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PhD Yearbook | 2021

PHD COURSE IN MATHEMATICAL MODELS AND METHODS IN ENGINEERING

Chair:
Prof. Irene M. Sabadini

Mathematics is everywhere, represented by equations. Between the atmosphere and the wing of a spaceship, in the blood flowing in an artery, on the demarcation line between ice and water at the poles, in the motion of the tides, in the charge density of a semiconductor, in the compression algorithms of a signal sending images from outer space. The equations represent real problems. The Mathematical Engineer can see and understand the nature of these equations, and can develop models in order to understand their relevant qualities and solve real problems.
This PhD program aims at training young researchers by providing them with a strong mathematical background and with ability to apply their knowledge to the solution of real-world problems that arise in various areas of science, technology, industry, finance, management, whenever advanced methods are required in analysis, design, planning, decision and control activities. PhD students carry our their research both in the development of new mathematical methods and in the implementation and improvement of advanced techniques in connection with specific contexts and applications.
The Faculty of the PhD program is responsible for the organization of the training and research activities of the PhD students. Decisions of the Faculty comply with the requirements and standards of the Doctoral School of the Politecnico di Milano. A Chairman is elected within the Faculty, for representative and coordination activities. Admission of students to the PhD program is decided after examination of the candidates. Students applying to our program must provide their CV, along with reference and motivation letters. After admission, each student is assigned a tutor. The tutor is a member of the Faculty who assists the student in the early stages of his career, especially in the choice of the courses and in identifying a thesis advisor.

The PhD program has a duration of three years. Activities include:
Soft skills courses; specialized courses; research training, including seminars, tutoring activity, participation to workshops/conferences, and scientific publications; development of a doctoral thesis.

At the end of each academic year, the PhD students report to the Faculty about their activity. The students report about attendance of courses and exams (and the corresponding grades), participation in various scientific activities (seminars, conferences, summer schools etc.), planning and intermediate results on their research project and preparation of the PhD thesis, and any other relevant activity. At the annual meeting the students also receive a grade by the Faculty. A negative grade may entail repetition of the current year of doctoral study (with suspension of the grant, if any) or exclusion from the PhD program, depending on the Faculty's decision. Mobility of PhD students to other institutions is strongly encouraged and financial support is provided to this purpose.

Among others, let us mention some typical types of professional skills and possible occupations of the graduated Doctors: analytic and numerical treatment of differential models for physical and industrial problems, quantitative methods in finance and risk management, operations research and optimisation, statistical modelling and data analysis.

Placement of graduated Doctors is expected in the following positions: research and development divisions of businesses, businesses involved in innovative design activities, financial institutions such as banks or insurance companies, public or private research centres, public and/or governmental agencies for social, economical, scientific study, planning or evaluation, Universities.

Since the PhD program in Mathematical Models and Methods in Engineering (formerly, Mathematical Engineering) has been active since the year 2001, we expect that a larger number of institutions and businesses will soon become more and more aware of the professional skills and expertise of graduated doctors.

Grants funded by external partners: ENI, IIT, Karman Institut

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MATHEMATICAL MODELS AND NUMERICAL METHODS FOR CARDIAC POROMECHANICS

Nicolas Alejandro Barnafi - Advisor: Alfio Quarteroni

Co-advisors: Paolo Zunino, Luca Dedè

Cardiac perfusion describes the heart's blood supply, which arrives through a complex network of vessels that surround it, known as coronary vessels. The mathematical modeling of this process involves the solution of complex multi-physics problems, which considers on one side the coronary vessels and on the other side the heart tissue, which is divided into the tissue itself and the porous structure induced by the ramifications of the vessels. An adequate framework for modeling such physics is that of nonlinear poromechanics, in virtue of two specific properties: the first one is that it is consistent with the mathematical theory of finite elasticity, which is fundamental for handling the deformation during a heartbeat. The second one is the use of rigorous averaging techniques which allow for computationally efficient models of the vessels, their interaction with the deforming tissue and the blood flow in them.

My research served two goals. The first one is the creation of an adequate framework for the development of robust numerical methods for the poromechanics problem; the second one is the construction of an efficient numerical strategy which embeds cardiac perfusion into a coupled full-heart simulation. We focused first on the analysis of a linearized problem, at both continuous and discrete levels. Its study reveals a saddle point structure in which the incompressibility constraint consists in a sum of the velocity of both fluid and solid phases, weighted by the material porosities. We show that the use of a Taylor-Hood type of finite elements is inf-sup stable, where interestingly the considered instabilities can coexist spatially, as seen in the pressure fields shown in Figure 1. As observed, the fluid phase is much more prone to instability, which can be quantified by computing the problem-specific inf-sup constant.

After this, we developed block-partitioned numerical schemes for the linearized problem which consist in iteratively decoupling the physics which compose the model. For this, we adapted well-known splitting schemes from Biot's consolidation model and provided the corresponding convergence analysis. More specifically, we focused on the undrained and fixed-stress schemes, which required very different approaches for their analysis. The former can be analyzed by means of the theory of generalized gradient flows, whereas the latter can be studied through the concept of R-linearly convergent subsequences. The convergence of the proposed splitting strategies is mesh independent, and in addition yields a solution strategy where at all times only the individual physics are solved. The sub-physics are easier to solve, and in a numerical study we observed that solution times where more than halved with our approach, with the advantage increasing with the size of the problem.
For the cardiac perfusion model, we considered the coronary vessels as a network of 0D elements, where we extended the existing lumped models to allow them to handle arbitrary combinations of boundary conditions, for which we showed their numerical properties and well-posedness. Classical 0D models require case-specific considerations for different combinations of boundary conditions (Dirichlet and Neumann), whereas our approach is capable of naturally handling all such cases plus linear combinations of them (Robin conditions) and their coupling for arbitrary networks of vessels (transmission conditions). Poromechanics models instead strongly depend on constitutive modeling. In this respect, we present a novel decomposition of the energy, which takes the form of a barrier function for the porosity. Then, we present a comprehensive comparison of nonlinear solvers for the poromechanics problem under consideration. By using the existing theory of nonlinear mechanics and porous media equations, we developed criteria for assessing the proposed model in terms of convexity (for the porous media) and the polyconvexity (mechanics) of the corresponding potentials, which results in guaranteeing the existence of solutions for the individual physics under consideration. Existence of solutions for the coupled problem remains an open problem.
Finally, we propose solving the coupled perfusion problem through a fixed point scheme, and test our methods on the realistic ventricle geometry provided by Zygote by decoupling model governing the mechanical deformation of the heart as shown in Figure 2.

Our results show that our model is able to adequately capture the physics of interest, where particular emphasis is given to the systolic impediment phenomenon, which describes the reduces blood inflow of the heart during systole, in strong contrast to the behavior of the rest of the organs in the human body.

Through our work, we provide guidelines for efficient and robust strategies for the numerical approximation of linear poromechanics, as well as provide promising preliminary results for the nonlinear scenario. We then propose a novel poromechanics modeling framework which is able to reproduce physiological conditions of a healthy heartbeat, and thus presents a powerful tool for the efficient creation of in-silico models.

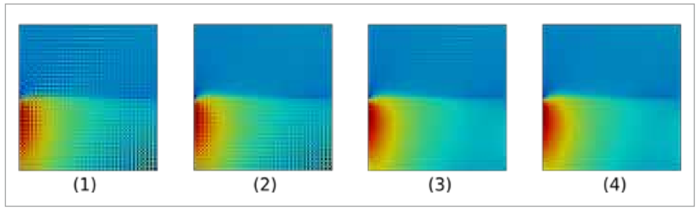

Fig. 1 - Dominating solid (above) and fluid (below) materials with (1) no stabilization, (2) only solid, (3) only fluid and (4) both stable.

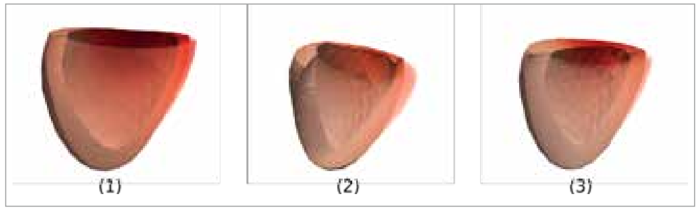

Fig. 2 - Evolution of blood in the capillaries during a heartbeat. (1) End-diastole, (2) systole and (3) diastole.

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SELF-ORGANIZATION AND PATTERN FORMATION IN SOFT MATTER

Giulia Bevilacqua - Advisor: Pasquale Ciarletta

This thesis deals with the formulation and analysis of mathematical models for soft matter. More precisely, we focus on two main phenomena: self-organization and pattern formation.

The first one is a spontaneous process which gives a specific order to a disordered system thanks to the multiple interactions of particles or parts of the system and due to the presence of an external energy which drives the entire process. Examples can be found in different subjects, like in Physics, Chemistry and Biology, but also in Economy, Sociology and Technology. For instance, we mention the protein folding, which drives the system to equilibrium configuration, or the different shapes of liquid crystals, which strongly depend on the amount of available energy.

Pattern formation refers to the generation of complex configurations organized in space and time, for instance the pigmentation of seashells or the skin of animals. In this thesis, we mainly focus on modeling growth and remodeling in living matter and studying possible consequences due to the presence of these active phenomena. For instance, in soft elastic solids, the mentioned active processes change the internal microstructure and cause geometrical incompatibilities, which, in combination with physical nonlinearities of the material itself, may induce a topological transition. Indeed, residual mechanical stresses, which are present even in the absence of external loads, can arise and once they exceed a critical threshold, an elastic instability can occur driving to a morphological change. Examples in this setting are the windy development of tumor growth in the healthy tissue or the formation of biological organs, like arteries and the intestine. Moving to a fluid-like description, the phenomenon of growth characterizes cancer development. Indeed, considering the tumor cell density evolving in the healthy tissue, modeled as a porous medium, to take into account its cell division rate, an additional source term is included to the model. Hence, classical results have to be modified since this new term can lead to different analytical solutions or, from a numerical point of view, the emergence of finger-like patterns.

The results of this thesis are collected int two chapters. In Chapter 2, we focus on two different problems to characterize the selforganization phenomenon in soft matter. First, we propose a theoretical explanation for the symmetry break in the arrangement of eight cells undergoing optimal compaction driven by anisotropies in the mechanical cues, mimic the mitotic process in the embryo passing from the eight-cell stage to the sixteen one. Second, we present recent extensions of the so-called Kirchhoff- Plateau problem, in which the fixed boundary of the classical Plateau problem is replaced by an elastic rod and it can be used as a prototype to study the process of absorption of a protein by a biological membrane.

In Chapter 3, we deal with different models to study growth and remodeling in living matter. Tammes' rearrangement First, we address the phenomenon of the gyrification, *i.e.* the formation of the folded structures in brain organoids by proposing an elastic model coupled with surface tension to correctly describe their experimental behavior. Second, we study the c-looping process in the heart tube, which is the first-symmetry breaking process in cardiac embryogenesis, by introducing an internal remodeling cell flow, and we show that this active process alone can drive the spontaneous onset and the fully nonlinear development of the c-looping.

Third, we propose a new model to characterize the onset of Faraday instability in soft tissues showing that standing waves at the free surface can appear also in soft elastic solids. By studying the linear problem through the Floquet theory, we obtain that Faraday instability in soft solids is characterized by a harmonic resonance, which will also introduce a new experimental procedure to distinguish a fluid-like from a solid-like response. Finally, we characterize growth process in a fluid-like system by studying cancer invasion. We consider the evolution of a tumor cell density through the healthy tissue, modeled as a porous-medium and, to take into account the cell division rate, we introduce an additional source function. Since it is known that models of this type are equivalent, in the incompressible limit, to more realistic tumor growth ones formulated as free boundary problems, we extend the Aronson-Bénilan estimate on the second derivatives for porous media in different Lebesgue spaces and for all fields of pressure.

All the aforementioned problems are solved within Mathematical Physics framework, dealing with the definition of a suitable model to characterize the real and observable physical phenomenon. Then, this model can be studied using different mathematical tools: we exploit variational formulations to look for an absolute minimum, perturbative techniques to describe the behavior in a neighborhood of an equilibrium point, energy estimates to control the evolution of a specific quantity or numerical methods to characterize the post-buckling behavior.

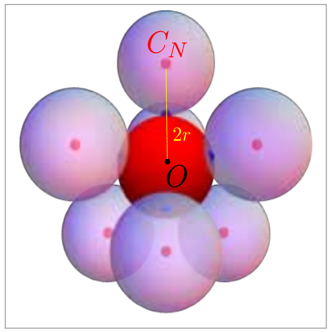

Fig. 1 - Initial arrangement of the bubbles before compaction according to Tammes' rearrangement

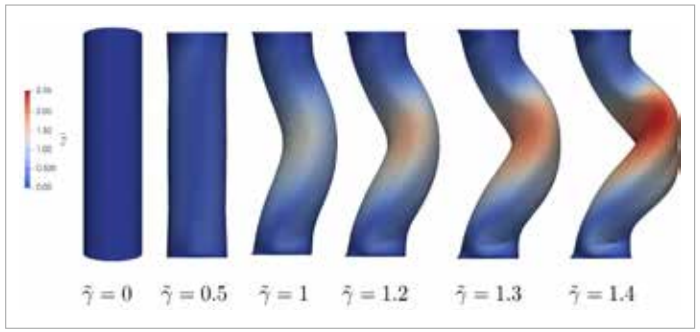

Fig. 2 - Actual configuration of the buckled tube using physiological values.

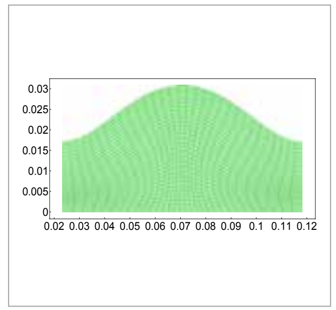

Fig. 3 - Morphology of the emerging Faraday wave in soft solid.

POPULATIONS OF GRAPHS: STATISTICAL ANALYSIS IN THE GRAPH SPACE WITH APPLICATIONS TO URBAN MOVEMENTS.

Anna Calissano - Supervisors: Simone Vantini, Valeria Fedeli

Populations of graphs are a complex and strongly non-Euclidean data type describing different relational phenomena in different fields such as brain connectivity networks of different patients, time series of mobility origin-destination matrices, users' social in different social platforms. The aim of this PhD thesis is to develop statistical tools for the analysis of populations of unlabelled graphs, embedding them in the Graph Space, a quotient space of permuted adjacency matrices. We perform cluster analysis, we define Geodesic Principal Components, and Graph-valued regression model. We introduce an algorithm, namely Align All and Compute, to estimate the defined intrinsic statistic in the Graph Space. These original statistical tools are applied to quantify and analyse urban movements, in order to understand how people move within a square, a city, a region. We discuss if the description of a spatial occurrence in an abstract space can reveal interesting perspectives about the analysis of reality.

A graph (or a network) is a mathematical structure used to study relational phenomena in different fields. It is characterized by a set of nodes and a set of relations between them. From a methodological perspective, the analysis of networks roots back in centuries and the literature has so far primarily focused on the analysis and modelling of a single network datum. Hereby, we focus on the analysis of a population of network-valued data: the analysis of not one but a set of networks. A population of network-valued data is a sample of networks describing a phenomenon. Network-valued data are an example of complex data and the statistical analysis of such data is generally referred to as Object-Oriented Data Analysis, a statistical stream of literature laying at the intersection between geometry and statistics.

The starting research question concerns the embedding space: the definition of a space where every complex datum (e.g. a network) is a point. If all the networks in the sample share the same nodes, the problem could be re-framed as an analysis of a set of matrices (i.e. adjacency matrices). This data type are called labelled networks. If the networks describe the same class of phenomena, but they have varying numbers of nodes or inconsistent node labelling, the mathematical challenges involved in the analysis become numerous. These data are called unlabelled networks and they are going to be the object of analysis in the current work. Among different embedding spaces, we choose Graph Space (a quotient space X/T obtained by applying permutation action T to adjacency matrices in the Euclidean space X) as a starting point. The embedding strategy and its geometrical characterization is detailed in the second chapter.

Once we defined the embedding strategy, the analysis of population of graphs required the definition of statistical tools such as Cluster Analysis, Fréchet Mean, and Geodesic Principal Components. Even if the Graph Space is not a manifold and its curvature is unbounded from above, we can take advantage of its relation to the Euclidean total space to define and compute intrinsic Graph Space statistics. Given the fact that Graph Space is a metric space, we start the third chapter by applying cluster analysis. Given the fact that Graph Space is a geodesic space, we study the properties of the Fréchet Mean and we extend tools such as Principal Components. However, geometrical characteristics of the space proven in the second chapter make unavailable the usage of the so called extrinsic methods in this context. Extrinsic methods - as tangent space statistics- have been broadly used as a solution for the definition and the computation of statistics in a non-Euclidean setting. Intrinsic methods instead define the statistics on the space itself. In the third chapter, we show that intrinsic Graph Space statistics can be computed iteratively via a combination of choosing optimal graph representatives in X/T, and computing Euclidean statistics in X. We call this iterative strategy Align All and Compute (AAC) and we will decline this general strategy to the Fréchet Mean, the Geodesic Principal Components, and the Regression in the last chapter.

The chapter contains several case study. To give a first idea about the data analysis in the graph space, we start by describing an urban planning case study: how the clustering of a population of simple graph can have an impact in the analysis of human movement in a square. To showcase principal components, we study the mobility in the Lombardy region as a set of multi-layers origin destination matrices.

The last chapter is about prediction methods. We introduce an intrinsic generalized geodesic regression for a graph-valued regression problem (model a graph from a vector or scalar). Inspired by the results obtained for Geodesic Principal Components in the previous chapter, the estimation of this regression function is done via Align All and Compute, which is declined hereby to minimize the prediction error. Along with the prediction model, a technique to estimate the uncertainty of the predicted value is required. The strongly non-Euclidean nature of these data type and the difficulties of inferring a distribution of a population of networks (especially unlabelled) creates the requirement of the definition of a non-parametric strategy. The second half of the last chapter focuses on the development of a conformal prediction interval for every prediction model for a population of graphs. Due to the complex covariance structure and the different phenomena on edges and nodes, making parametric inference hard to apply in complex-valued data. We thus define a non parametric conformal prediction strategy, defined for both labelled and unlabelled networks. The last chapter contains two case studies about the effect of 2020 Covid-19 pandemic outbreak on the urban and the territorial mobility. The first one regards prediction of the public transport network usage in Copenhagen during Covid-19 is studied, analysing with an anova model the effect of the lockdown on the networks dynamics. In the second one, we analyse the Origin Destination Matrices of Lombardy region before, during, and after the total lockdown, understanding the distribution of the fluxes between the different provinces.

This thesis focuses on the statistical analysis of population of graphs and its application and potential impact in the analysis of urban movement. It offers a complete overview of different steps required to explore a non-euclidean valued dataset, starting from the definition and description of an embedding space, moving towards the introduction of novel methodology and computational strategies aimed at analysis the data in the natural space. Along the mathematical modelling, the thesis poses open research questions and proposes case studies regarding the possible usage of non-euclidean statistics and non-euclidean spaces in the analysis of urban phenomena.

A MULTI-PHYSICS MATHEMATICAL AND NUMERICAL MODEL FOR THE SIMULATION OF MYOCARDIAL PERFUSION

Simone Di Gregorio - Supervisor: Alfio Quarteroni

Co-supervisors Christian Vergara, Paolo Zunino

The identification of coronary artery stenosis and associated ischemia is of utmost importance in the identification of patients who should be addressed to further invasive evaluation and revascularization.

In clinical practice, the quantification of the myocardial blood flow (MBF) and the functional assessment of coronary artery disease (CAD) is achieved through the post processing of stress myocardial computed tomography perfusion (stress-CTP). Despite the promising results, stress-CTP requires an additional scan on top of coronary computed tomography angiography (cCTA) and an intravenous stressor administration. Computational methods could be used in this context to provide a non-invasive assessment of the MBF minimizing the biological burden.

In this work we proposed a novel mathematical and numerical multi-physics model for cardiac perfusion which accounts for the different length scales of the vessels in the coronary arterial tree.

Epicardial coronary arteries were represented with fully three-dimensional fluid-dynamics, whereas intramural vessels were modeled as a multi-compartment porous medium. The coupling of these models took place through interface conditions based on the continuity of mass and momentum. The design of the multi-physics model is displayed in Figure 1.

We also presented efficient numerical solvers for the coupled problem. In particular, we proposed a splitting algorithm for the coupled problem, with the corresponding convergence analysis performed in a linearized case, and a suitable preconditioner for the multi-compartment porous sub-model.

In order to obtain accurate results, we estimated proper model parameters. We proceeded in two steps. The first step used only information from cCTA and a surrogate intramural vascular network, which was generated with a novel algorithm working also in non-convex domains. The second step exploited the patient-specific MBF maps measured from stress-CTP to optimize the parameters and to improve the accuracy of the model.

Finally, we applied the computational framework to patient-specific scenarios and we assessed the reliability of the computational multi-physics model in reproducing patient-specific MBF maps. Specifically, we consider nine patients (four healthy and five with detected CAD), whose clinical data are provided by the Monzino Cardiology Centre. In Figure 2 the results for two selected patients are shown. It is possible to notice an excellent agreement between the MBF computed in the numerical simulations and the MBF measured starting from the stress-CTP.

The accuracy and the reliability of the proposed computational model in reproducing MBF represent a first necessary step towards the construction of a computational tool able to predict MBF avoiding the stress protocol, thus preventing potential stressor's related side effects and reducing the radiation exposure.

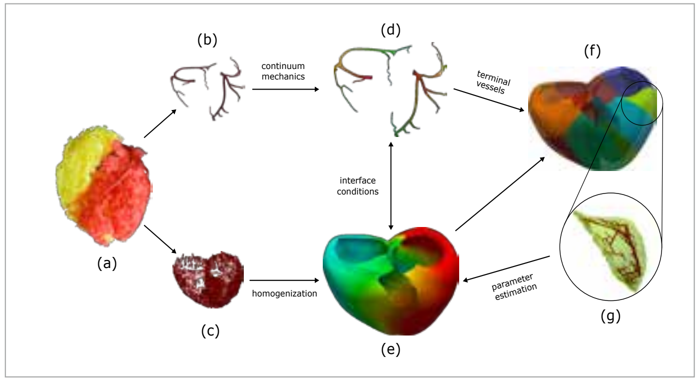

Fig. 1 - Design of the multi-physics model: (a) Cast of Coronary Arteries, source Wikipedia; (b) epicardial coronary arteries; (c) intramural coronary vessels; (d) 3D blood flow dynamics model inside coronary domain (pressure depicted); (e) porous media flow model inside myocardial domain (pressure depicted); (f) myocardium partitioned into different perfusion regions; (g) example of generated intramural vascular network inside a perfusion region.

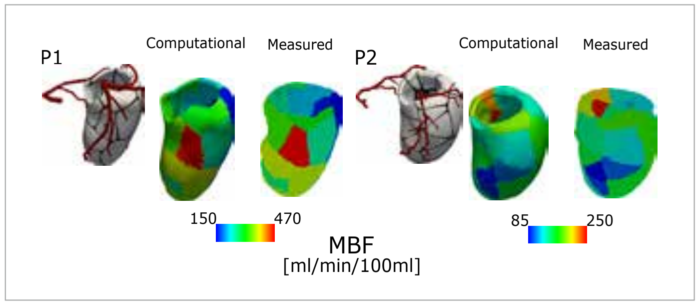

Fig. 2 - Comparison between computational results of MBF (left) and clinically measured MBF (right) in two patients. On top-left the perfusion regions with epicardial coronary arteries.

REPRODUCING KERNEL TECHNIQUES AND POLYANALYTIC FUNCTION THEORY IN HYPERCOMPLEX ANALYSIS

Kamal Diki - Supervisor: Irene Maria Sabadini

In this PhD thesis, we study different reproducing kernel function spaces and associated integral transforms in the setting of complex, quaternions and Clifford analysis. In particular, we focus on some specific examples such as Segal-Bargmann-Fock spaces, Bergman spaces, Hardy spaces and Gabor spaces. These models are very important in complex analysis, operator theory and have several applications in mathematical physics, time frequency analysis and signal processing, especially in quantum mechanics.
As it is well-known, in quantum mechanics physical quantities such as position, momentum and energy are represented by operators acting on some complex Hilbert spaces. In 1961, Bargmann constructed a Hilbert space of entire functions on which the creation and annihilation operators are adjoints of each other's and satisfy the classical commutation rules. This space is known as Fock or Segal-Bargmann space, sometimes called also the bosonic Fock space with n degrees of freedom. To any particle moving on the real line is associated a wave function which defines a unit vector of the classical Hilbert space on the real line. This unit vector is mapped onto a special holomorphic function making use of a particular exponential kernel. The new resulting complex function is the so-called Segal-Bargmann transform.
Another interesting contribution that we achieved during this research project is that we initiated exploring a new research path by extending the theory of slice regular or slice monogenic functions to higher order and considering the so-called slice poly-analytic or slice poly-monogenic function theory on which several questions are open now. We note also that a generalized version of the poly Cauchy formula and the famous Fueter-Sce-Qian mapping theorem in Clifford analysis were also introduced and proved in this framework. This new construction allowed to relate different poly function theories in hypercomplex analysis.
Furthermore, an important fact that was observed is that this slice polyanalytic function theory contains one of the most important subclasses of the Cauchy-Fueter hyperholomorphic function theory, namely the class of Fueter hyperholomorphic functions of axial type. A very natural and interesting problem that has to be considered now is to develop a natural S-functional calculus associated to this new poly-analytic function theory. In this research project, under the supervision of Prof. Irene Maria Sabadini, I co-authored 8 original research papers, from which four are already published, three are accepted to appear and one is still submitted. We give a brief overview on the different results obtained in this PhD thesis:

- The Banach Fock spaces of slice hyperholomorphic functions on the quaternions were introduced, both of the first and of the second kind. In particular, we proved several approximation results on these different spaces, some of them are based on constructive methods making use of the Taylor expansion and the convolution polynomials. The techniques used in these two cases are different. Moreover, for the second kind theory, we discussed also some density results of reproducing kernels.
- A new definition extending to higher order the theory of slice hyper-holomorphic functions on the quaternions originally introduced by Gentili and Struppa in 2007 was proposed. This definition extends the notion of complex polyanalytic functions to quaternions. We studied some basic properties of such functions and presented some of their consequences. Then, we considered the Fock and Bergman spaces in this new setting and computed explicit expressions of their reproducing kernels.
- A Cholewinski-Fock space in the slice hyperholomorphic setting was studied. It presents an extension of the classical slice hyperholomorphic Fock space introduced in 2014 by Alpay, Colombo, Sabadini and Salomon. This was possible by considering on the space of slice entire functions a specific weight involving a modified Bessel function of the third kind, namely the Macdonald's function. We gave a complete description of this quaternionic Hilbert space. Then, its reproducing kernel is obtained making use of the slice hyperholomorphic extension of the classical complex Dunkl kernel. We introduced also an associated unitary integral transform, and study some specific quaternionic operators on the slice hyperholomorphic Cholewinski-Fock space. This construction follows an approach by Cholewinski in 1984.
- Some applications of the famous Sce-Qian-Fueter mapping theorem were investigated. More precisely, making use of the Sce-Qian-Fueter mapping theorem we constructed and studied some special integral transforms of Bargmann-Fock type in the setting of quaternion slice hyperholomorphic and Cauchy-Fueter regular functions. In particular, starting with the normalized Hermite functions we got a Clifford-Appell system of quaternionic regular polynomials.
- A new quaternionic short-time Fourier transform QSTFT with a Gaussian window was introduced. We proved several results about this QSTFT like a Moyal formula, a reconstruction formula and a Lieb uncertainty principle. This construction was possible thanks to the use of the quaternionic Segal-Bargmann transform. Moreover, we computed the reproducing kernel associated the Gabor space considered in this framework.
- The slice polyanalytic functions of order n on quaternions were considered also as null solutions of the n-th power of a certain global operator with non-constant coefficients as it happens in the case of slice hyperholomorphic functions. We investigated also an extension version of the Fueter mapping theorem in this polyanalytic setting. In particular, we showed that under axially symmetric conditions it is always possible to construct Fueter regular and poly-Fueter regular functions through slice polyanalytic ones using what we call the poly-Fueter mappings. Furthermore, we proved a new poly-Cauchy formula that suggests to start several new interesting research problems. As a first application of this poly-Cauchy formula we gave an integral representation of the poly-Fueter mapping theorem, extending a very important result obtained in 2010 by Colombo, Sabadini, Sommen.
- A specific system of Clifford-Appell polynomials and in particular their CK-product were considered. We first studied how this system behave with respect to the CK-product. We gave also a characterization of Fueter hyperholomorphic functions of axial type in terms of this system. Then, we introduced a new family of quaternionic reproducing kernel Hilbert spaces in the framework of Fueter regular functions. This construction is based on a general idea which allows to obtain various function spaces, by specifying a suitable sequence of real numbers. We focused more on the Fock and Hardy cases and we studied the action of the Fueter mapping and its range.
- A new research was intiated to begin the study of Schur analysis and de Branges-Rovnyak spaces in classical quaternionic and Clifford analysis, in particular in the framework of Fueter hyperholomorphic functions. We treated there several problems related to Hardy space, Schur multipliers, Blaschke functions, Herglotz multipliers and their associated kernels and Hilbert spaces.

DEEP LEARNING-BASED REDUCED ORDER MODELS FOR NONLINEAR PARAMETRIZED PDES: APPLICATION TO CARDIAC ELECTROPHYSIOLOGY

Stefania Fresca - Advisor: Alfio Quarteroni

The electrical activation of the heart, which drives its contraction, is described by a system of partial differential equations (PDEs) suitably coupled with ordinary differential equations (ODEs). Solving this system by a full order model (FOM) entails prohibitive computational costs when its dimension is large. Even more challenging is the repeated solution of these equations, for evaluating different outputs of clinical interest, such as activation maps (ACs) and action potentials (APs), corresponding to different inputs.
Reduced order modeling techniques aim at replacing the FOM by a reduced order model (ROM) featuring a much lower dimensionality but still retaining the physical features of the phenomena described by the FOM. The main assumption behind a ROM is that the solutions of a parametrized PDE, belonging to a high-dimensional discrete space, lie on a low-dimensional manifold embedded into the high-dimensional space. The goal is then to approximate the solution manifold through a suitable trial manifold. Projection-based ROMs, approximate the solution manifold by a reduced linear trial manifold. Then the ROM solution is a linear combination of basis functions computed, for example, by means of proper orthogonal decomposition (POD). Models featuring coherent structures that propagate over time, such as the equations describing the electrical activation of the heart, are not amenable to classical projection-based reduced order modeling strategies, such as the reduced basis (RB) method and its local counterpart, due to their inability to accurately approximate the solution manifold by a low-dimensional linear subspace. Moreover, such kind of ROMs must account for the dynamics of the gating variables as well, even if one is interested in approximating just the electrical potential. This fact entails an extremely intrusive and costly hyper-reduction stage to reduce the solution of the ODE system to a few, selected mesh nodes.

Motivated by the necessity to overcome the limitations of linear ROMs we developed a new, non-intrusive technique based on DL algorithms to build efficient ROMs for parametrized PDEs. The dimension of the approximated manifold is nearly equal (if not equal) to the dimension of the solution manifold. The resulting DL-ROM learns both the reduced nonlinear trial manifold in which the ROM solution is sought, and the nonlinear reduced dynamics on the approximated manifold, providing in such a way the ROM solution starting from a set of FOM solutions obtained for different input instances. The nonlinear trial manifold is learnt by means of the decoder function of a convolutional autoencoder (AE) neural network, whereas the reduced dynamics through a (deep) feedforward neural network (DFNN), and the encoder function of the convolutional AE (see Fig. 1).
We show that DL-ROMs outperform

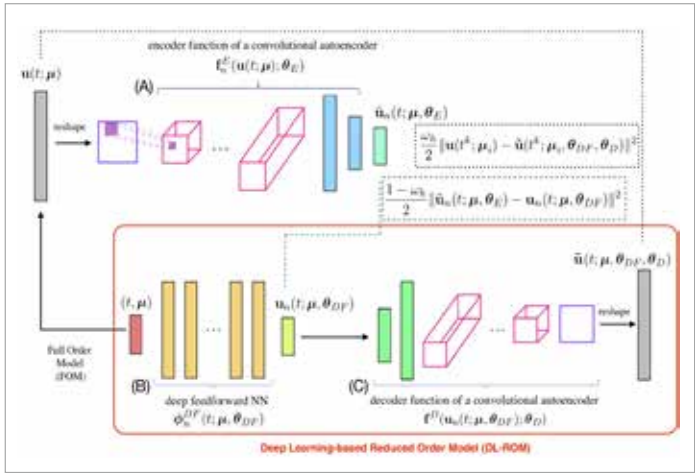

Fig. 1 - DL-ROM architecture.

linear ROMs, such as, e.g., POD-Galerkin ROMs – even involving local reduced bases – regarding both numerical accuracy (for the same ROM dimension) and computational efficiency during the online (or testing) stage, when applied to problems that are typically challenging for the RB method (such as, e.g., linear transport equation, nonlinear diffusion-reaction equations whose solution develops moving fronts depending on parameters).

We focused on the enhancement of the computational efficiency of DL-ROMs during the offline (or training) stage, which is also strongly related with the curse of dimensionality. In particular, a strategy is proposed to enhance DL-ROMs in order to make the offline training stage dramatically faster, allowing for much larger FOM dimensions, without affecting the number of networks parameters to be estimated and, ultimately, network complexity. This strategy exploits (i) dimensionality reduction of FOM snapshots through randomized POD (rPOD), to be considered as the action of the first layer of the convolutional AE, rather than the way to generate the linear trial manifold, as done instead in traditional POD-Galerkin ROMs, and (ii) a suitable multi-fidelity pretraining stage, where different models (built, e.g., by considering coarser discretizations or simplified physical models) can be efficiently combined, to iteratively initialize the network parameters. These substantial enhancements of the DL-ROM technique provide a new way to build DL-based ROMs, which we refer to POD DL-ROM.
The performance of the POD DL-ROM technique is assessed on relevant test cases in cardiac EP on realistic geometries, both in physiological and pathological scenarios and considering snapshots arising from both finite element method and NURBS-based Isogeometric Analysis discretizations, as shown in Fig 2. These examples demonstate to be remarkable challenging tasks for reduced order modeling due to the steep wavefronts, the complex activation patterns associated to pathological scenarios, the high FOM dimension and the complexity of the geometries. (POD) DL-ROMs show to yield accurate and extremely efficient numerical approximations. This is particularly useful in view of the evaluation of patient-specific features to enable the integration of computational methods in current clinical practice; indeed outputs of clinical interest, such as ACs and APs can be more efficiently evaluated by the POD DL-ROM than by a FOM, while maintaining a high level of accuracy.

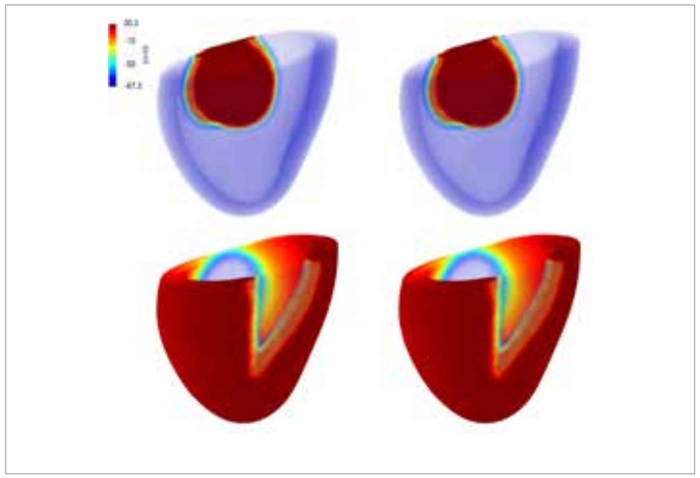

Fig. 2 - FOM and POD DL-ROM solutions on the 3D Zygote template left ventricle geometry.

THREE-DIMENSIONAL PHYSICS-BASED NUMERICAL SIMULATIONS OF EARTHQUAKE GROUND MOTION FOR ADVANCED SEISMIC RISK ASSESSMENT

Laura Melas - Advisor: P.F. Antonietti

Co-advisor: I. Mazzieri

In this thesis we aim at proposing and analyzing new coupled numerical models for seismic risk assessment. We develop a method that couples physics-based differential models for ground motion prediction with prediction models of structural damage. To predict the ground motion induced by an earthquake we employ a differential model based on the elastodynamics equations discretized by discontinuous Galerkin spectral element methods. This model is then suitably coupled with suitable models to quantify the seismic risk on structures that are based on employing either suitable vulnerability models, (e.g. fragility curves) or the structural analysis. This coupled approach yields an estimation of the predictable damage induced by an earthquake. This information allows us, for example, to quantify the real level of risk, and can guide at reducing the uncertainty related to two of the most important components of seismic risk: the hazard and vulnerability.
More precisely, in the first proposed coupled model, we deal with an approach that combines physics-based elastodynamics differential model for ground shaking prediction with the use of fragility curves for the risk assessment of structural vulnerability. In this way using a deterministic tool for earthquake ground motion prediction based on physical equations rather than empirical methods, we can obtain a more accurate information to assess ground motion hazard. As a result the input for fragility curves could become more reliable and as a consequence the corresponding predictions of possible structural failures. In general, the fragility function is defined as the conditional probability of a given damage state exceeding a threshold, given a value of the ground motion intensity measure. In Figure 1 we show the profiles of fragility curves, i.e., $P(DS \geq D_S|SD)$ versus SD, for high-rise buildings, where the spectral displacement SD is the earthquake intensity measure, DS is the damage state variable that measures the damages of a construction in a qualitative way and each colour denotes the type of damage state D_S. The proposed method is tested considering the metropolitan Beijing high seismicity urban area. Based on employing our model, we produce maps of seismic damage focusing on the specific class of high-rise buildings, accounting for a wide set of fault rupture realizations with magnitude in the range 6.5-7.3 Mw.

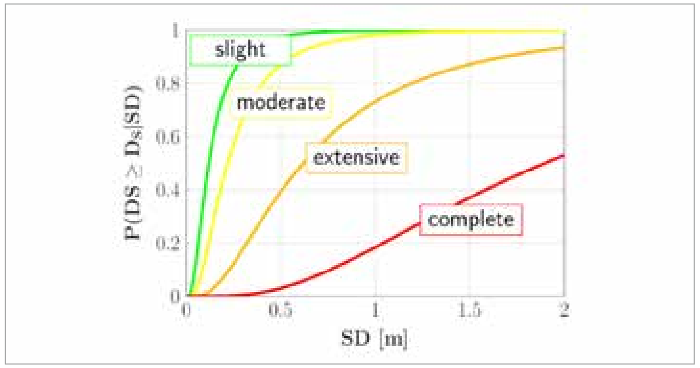

Fig. 1 - Fragility functions for high-rise buildings in Beijing.

Then, we propose a second fully physics-based approach by considering the differential models for modelling both the earthquake phenomenon and the seismic response of buildings. In particular, with this three-dimensional physics-based model, it is possible to take into account the interactions of the structures with respect to the earthquake ground motion within a multi-scale simulation. In such a way, the measure of the seismic risk level of a building is more accurate. The proposed model is validated considering a “source-site-structure” simulation of the 1999 Mw6 Athens earthquake to investigate the seismic response of the Parthenon, the main cultural heritage of Greece, and the results are compared with recorded data. In Figure 2 it is represented the 3D model of the Parthenon on the top of the Acropolis hill.

In order to correctly model the earthquake phenomenon, it is important to model several geophysical features, among them there are the topography, material properties of the layers, the geometry of the fault, the magnitude and the seismic source representation. In particular, the seismic source can be described through a kinematic approach by prescribing the slip along the fault or with a dynamic source modeling by introducing physical laws for earthquake spontaneous fault rupture. In the first part of this thesis we adopted a kinematic description, but we are interested in improving the seismic source within its dynamic representation. The advantage of the dynamic source modelling is that it physically describes a phenomenon in a more realistic way, but as a drawback it is much more computationally challenging due to the underlying governing equations. Moreover the contact-friction laws that are at the basis of the dynamic fault rupture modelling can be employed to describe the non-linear behaviour of the buildings within the framework of the structural analysis. Within this framework, efficiency of the numerical methods is very important. Motivated by the applicative framework, we develop new algebraic multigrid (AMG) methods for high-order discontinuous Galerkin (DG) discretizations. We proposed a new class of AMG methods for the efficient solution of the linear system of equations stemming from high order DG discretizations of second-order elliptic problems. The key observation is that, for nodal DG methods, standard multigrid approaches cannot be employed because of redundancy of the degrees of freedom associated to the same (physical) grid point. To overcome this limitation, we propose new aggregation algorithms. The proposed algebraic multigrid method is numerically demonstrated to be uniformly convergent with respect to all the discretization parameters, namely the mesh-size and the polynomial approximation degree.

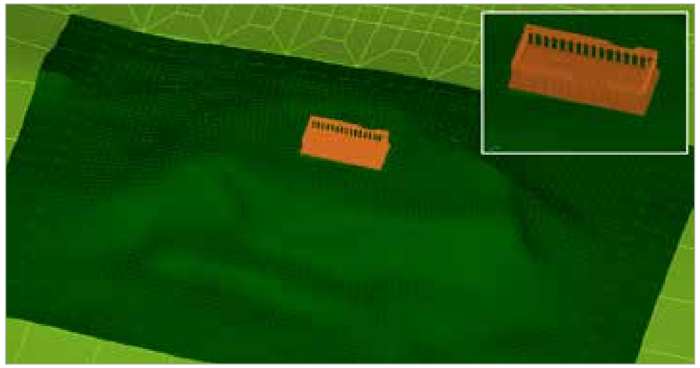

Fig. 2 - 3D computational model of the Acropolis hill and the Parthenon.

VARIATIONAL SOLUTIONS OF THE LINEARIZED BOLTZMANN EQUATION FOR THERMAL-DRIVEN GAS FLOWS IN MICROCHANNEL

Nhu Ngoc Nguyen - Supervisor: Silvia Lorenzani

Over the last decades, micro-electro-mechanical system (MEMS) devices developed rapidly and found many applications in microelectronics, chemistry industries, mechanical engineering, biology, medicine, vacuum technology and in other high technology fields. In particular, thermal MEMS (like thermal accelerometers, thermal flow sensors, thermal actuators, hot-wire anemometers) are gaining popularity both in commercial applications and in scientific research. In all these devices the working mechanism is related to the existence of temperature gradients. The *thermal transpiration flow i*s one of the classical problems in kinetic theory and it has been studied since 1879 when Reynolds has discovered this phenomenon. He defined the thermal transpiration as the flow through a gas-filled porous plate caused by a temperature difference between the two sides of the plate. In 1924, Hettner published a theoretical paper on the thermal transpiration problem and used the term “thermische Gleitung”, translated as “thermal creep”.
Micro- and nanodevices are often operated in gaseous environments (typically air), and thus their performance is affected by the gas around them. Since the smallest characteristic length of MEMS is comparable with (or smaller than) the mean free path of the gas molecules, the traditional computational fluid dynamics methods, based on the Euler or the Navier-Stokes equations, do not allow to predict the flows related to these devices. Therefore, an accurate analysis of such microfluidic systems requires the solution of the *Boltzmann equation*, which describes the evolution of the velocity distribution function of the gas molecules in non-equilibrium statistical mechanics.
In spite of the apparently complex structure, the basic constituent of a real MEMS device is the *microchannel,* the region between two parallel plates that can reveal many specific features of the low-speed internal flows in microdevices. Thus, an important aspect of the matter is to have an approximate closed form solution for gas flow rates in microchannels in order to use it in applications. To develop an accurate formula directly from the kinetic theory, there is a particularly useful technique, the variational method proposed by Carlo Cercignani in 1969, which applies to the integrodifferential form of the Boltzmann equation and can be used for any linearized Boltzmann model. On a different ground, the variational principles introduced previously apply to the integral form of the linearized Boltzmann equation, which is available explicitly only for simplified kinetic models. Compared to the numerical solutions of the kinetic equations which are computationally demanding, the variational approach has the enormous advantage of reducing drastically the computational CPU time giving at one time the same accuracy for each value of the Knudsen number.
The aim of this dissertation is to provide analytical/semi-analytical solutions of the linearized Boltzmann equation in microchannels. In particular, the variational method proposed by Carlo Cercignani in 1969 is proved to be a particularly useful technique for writing down simple approximate closed-form solutions of the Boltzmann equation in order to use them in applications. Since, so far, the effect of thermal gradients on MEMS devices has not been extensively investigated, in this thesis, the temperature-driven (thermal creep) gas flows are analyzed in the whole range of the Knudsen number (defined by the ratio of the molecular mean free path to a characteristic length of the system).

We have applied the variational method directly to the integrodifferential form of the Boltzmann equation based on a simplified kinetic model, the Bhatnagar, Gross, and Krook (BGK) model and on the true linearized collision operator for hard-sphere molecules. General boundary conditions (the Maxwell model and the Cercignani-Lampis scattering kernel) have been considered to take into account different types of molecular interactions between gas-solid interfaces.

In the case of low rarefaction level, the gas flow can be still simulated in the frame of the continuum modeling based on the Navier-Stokes-Fourier system. However, the implementation of special boundary conditions is necessary to take into account the rarefaction effects. These conditions involve the thermal-slip coefficients which depend on the gas-solid surface interaction through the accommodation coefficients.
A second-order slip model for the temperature-driven (thermal-creep) mass flow rate has been proposed on the basis of the variational analysis. Especially, in the case of the true linearized Boltzmann collision operator for hard-sphere molecules and the Cercignani-Lampis boundary conditions, the variational results for the first- and second-order thermal slip coefficients have been compared with recent experimental data for five noble gases (Helium, Neon, Argon, Krypton and Xenon) and, for each of them, the accommodation coefficients have been extracted. Then, these values of the accommodation coefficients have been used to evaluate the temperature-driven mass flow rates in the frame of our variational analysis and the outputs have been compared with the measurements for Helium, Neon and Argon. The good agreement obtained between the theoretical and the experimental data, within the range of validity of the proposed second-order slip model, suggests that the Cercignani-Lampis boundary conditions, unlike the Maxwell model, can conveniently be used to describe non-isothermal gas flows. For all the gases analyzed, the tangential accommodation coefficient is found to be much larger than the normal energy coefficient. The general trend, according to which, by increasing the molecular weight of the different gases, the values of both accommodation coefficients also increase, is confirmed in our study.

IMAGE-BASED FLUID-STRUCTURE INTERACTION MATHEMATICAL MODELS FOR THE SIMULATION OF ATHEROSCLEROSIS

Silvia Pozzi - Advisor: Christian Vergara

Co-advisors: Paolo Zunino, Alberto Redaelli, Emiliano Votta

Atherosclerosis is a vascular disease affecting the arterial wall, leading to the chronic inflammation of its inner layers and to the development of an atherosclerotic plaque or atheroma. As one of the main contributors to morbidity and mortality worldwide, alongside other cardiovascular diseases, it represents an active field of interest in the scientific community, with the objective of understanding and quantifying the complex inflammatory, immunological and biochemical processes that lead to plaque formation, growth, and vulnerability.

In light of these considerations, computational approaches aided by the use of medical imaging, referred to as image-based computational modeling, have emerged as tools to study atherosclerosis. To this aim, in this thesis novel mathematical and computational models applied to the study of atherosclerosis were proposed. In particular, two approaches were developed to study the processes leading to the growth of plaque and its influence on the hemodynamics in carotid arteries.

The first objective of this thesis was the development of a surrogate model, based on fluid-structure interaction, able to accurately describe the influence of the atherosclerotic plaque on blood dynamics in carotid arteries. This model is based on substituting the three-dimensional plaque with a set of independent springs, applied at the external surface of the stenotic carotid wall, thus surrogating the presence of the atheromatous tissue through the prescription of a suitable Robin boundary condition for the structure problem. To improve the accuracy of the model, the deformable constraint exerted by the adjacent jugular vein was also characterized adopting the same strategy (Fig. 1, left).

Clinical imaging was integrated with the computational model to generate patient-specific carotid geometries from three-dimensional MRI images and obtain inlet boundary conditions from color Doppler echocardiography measurements. Moreover, a specific processing pipeline was developed to obtain in vivo lumen-wall boundary displacements from CINE MRI acquisitions. This procedure allowed to obtain reference displacements with a sub-pixel resolution, which were used to estimate a subset of model parameters.

This approach was applied to three subjects featuring a stenosis of at least 70%. In all cases, accounting for both the plaque and the jugular vein improved the agreement of numerical results with CINE MRI data (Fig. 1, right). The surrogate FSI model proved comparably accurate with respect to the full FSI model, in which the plaque is modeled as a three-dimensional component, particularly when compared with the corresponding fixed-wall setting. Furthermore, the surrogate FSI model does not require the time consuming and often uncertain segmentation of the 3D plaque from medical imaging, thus making it attractive for use in a real clinical setting to study fluid dynamics in presence of plaque.

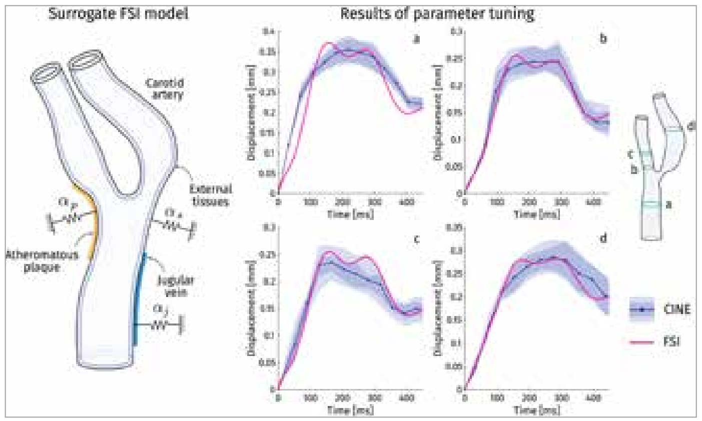

Fig. 1 - Schematic representation of the surrogate FSI model and sample of the obtained results.

The second objective of this thesis was the development of a mathematical and numerical model able to describe the initiation and early growth of the atherosclerotic plaque. This model couples processes with different spatial and temporal scales, as it is based on the interaction between macroscopic cardiovascular processes and molecular and cellular events typical of atherogenesis.

The multiscale-in-space nature of the process of plaque growth was treated by coupling a fluid-structure interaction problem, arising between blood and the arterial wall, to a set of differential problems describing the evolution of a subset of selected key species, i.e. LDL, macrophages, and foam cells. At the numerical level, a strategy based on the splitting and sequential solution of the coupled problem was adopted to deal with the multiscale-in-time nature of the system. The coupling between the two sub-problems was taken into account by introducing a two-way feedback between micro and macro scales, based on the adoption of a law linking time-averaged wall shear stress to endothelial permeability on one hand, and a growth law on the other.

The model was tested in ideal geometries and in real carotid arteries, both healthy and diseased, to assess its ability to produce significant plaque growth (Fig. 2). Additionally, the effects of the initial geometry, of a subset of model parameters and of different coupling timings on plaque growth were tested. In particular, the frequency of geometric update due to growth influenced the morphology and volume of plaque as much as the choice of parameters, thus highlighting the importance of employing a suitable multiscale-in-time strategy.

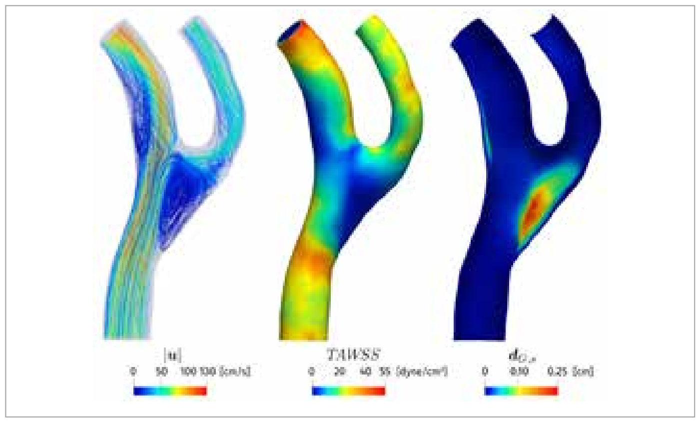

Fig. 2 - Velocity streamlines, distribution of time-averaged wall shear stress (TAWSS) and obtained plaque growth in a selected patient.