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PhD Yearbook | 2020

PHD COURSE IN MATHEMATICAL MODELS AND METHODS IN ENGINEERING

Chair:
Prof. Irene M. Sabadini

Mathematics is everywhere, represented by equations. Between the atmosphere and the wing of a spaceship, in the blood flowing in an artery, on the demarcation line between ice and water at the poles, in the motion of the tides, in the charge density of a semiconductor, in the compression algorithms of a signal sending images from outer space. The equations represent real problems. The Mathematical Engineer can see and understand the nature of these equations, and can develop models in order to understand their relevant qualities and solve real problems.
This PhD program aims at training young researchers by providing them with a strong mathematical background and with ability to apply their knowledge to the solution of real-world problems that arise in various areas of science, technology, industry, finance, management, whenever advanced methods are required in analysis, design, planning, decision and control activities. PhD students carry our their research both in the development of new mathematical methods and in the implementation and improvement of advanced techniques in connection with specific contexts and applications.
The Faculty of the PhD program is responsible for the organization of the training and research activities of the PhD students. Decisions of the Faculty comply with the requirements and standards of the Doctoral School of the Politecnico di Milano. A Chairman is elected within the Faculty, for representative and coordination activities. Admission of students to the PhD program is decided after examination of the candidates. Students applying to our program must provide their CV, along with reference and motivation letters. After admission, each student is assigned a tutor. The tutor is a member of the Faculty who assists the student in the early stages of his career, especially in the choice of the courses and in identifying a thesis advisor.

The PhD program has a duration of three years. Activities include:
Soft skills courses; specialized courses; research training, including seminars, tutoring activity, participation to workshops/conferences, and scientific publications; development of a doctoral thesis.

At the end of each academic year, the PhD students report to the Faculty about their activity. The students report about attendance of courses and exams (and the corresponding grades), participation in various scientific activities (seminars, conferences, summer schools etc.), planning and intermediate results on their research project and preparation of the PhD thesis, and any other relevant activity. At the annual meeting the students also receive a grade by the Faculty. A negative grade may entail repetition of the current year of doctoral study (with suspension of the grant, if any) or exclusion from the PhD program, depending on the Faculty's decision. Mobility of PhD students to other institutions is strongly encouraged and financial support is provided to this purpose.

Among others, let us mention some typical types of professional skills and possible occupations of the graduated Doctors: analytic and numerical treatment of differential models for physical and industrial problems, quantitative methods in finance and risk management, operations research and optimisation, statistical modelling and data analysis.

Placement of graduated Doctors is expected in the following positions: research and development divisions of businesses, businesses involved in innovative design activities, financial institutions such as banks or insurance companies, public or private research centres, public and/or governmental agencies for social, economical, scientific study, planning or evaluation, Universities.

Since the PhD program in Mathematical Models and Methods in Engineering (formerly, Mathematical Engineering) has been active since the year 2001, we expect that a larger number of institutions and businesses will soon become more and more aware of the professional skills and expertise of graduated doctors.

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CONSERVATIVE MULTIRATE SCHEMES FOR FLOW IN HETEROGENEOUS POROUS MEDIA

Ludovica Delpopolo Carciopolo - Advisor: Prof. Luca Formaggia

Computational models of multiphase flows in porous media are of great importance in many areas of engineering – e.g., hydrology and groundwater flows, oil and gas reservoirs, geothermal energy, and waste management, including CO_2 sequestration in the subsurface.
In hydrology and groundwater applications, the quality of the water is often endangered by a large number of contaminants (e.g., atmospheric contaminants, leakage from storage tanks that may contain gasoline, oil or chemicals). Multiphase flow models allow us to predict the flow of these contaminants, which may form a separate phase immiscible with water, by means of suitable equations.
Regarding oil reservoirs exploitation, an important technique for Hydrocarbon extractions is EOR – Enhanced Oil Recovery. EOR methods are based on the injection of different substances (thermal, gas, or chemicals) in the reservoir in order to increase the recovery factor of the oil. It is crucial to develop high-fidelity simulations of EOR in order to operate efficiently these advanced techniques.
One of the most relevant application of multiphase flow is geothermal energy production, which is strictly linked with renewable energy. The thermal energy is contained in the rocks and fluids from the shallow ground to several miles below the surface. Indeed, these underground reservoirs can be topped to generate electricity or used to directly heat and cool buildings. The energy can be extracted directly from the groundflow, with no combustions and nearly zero air emissions. This complex scenario can be modeled by the equations of multiphase flow to understand the phenomena and increase the amount of green energy production.
Finally, CO_2 sequestration is a process used to capture and store the atmospheric carbon dioxide (CO_2). This carbon dioxide is the product of industrial processes and it can be stored in geological formations, such as subsurface saline aquifers, reservoirs, or coal seams. Moreover, the captured CO_2 can be also used for EOR to displace oil. Numerical methods are very important to study the physical mechanisms related to CO_2 sequestration, in particular to assess the safety of long-term storage by evaluating the risk of leakage.

All these geoengineering applications involve a number of physical phenomena that take place at different scales, from the micro-scale scale, where localized alteration of the rock properties may occur, to the large reservoir scale where the fluids displace. In fact, on one hand geological formations extend for several hundreds of meters; on the other hand, physical and chemical phenomena, which are of interest for the described applications, occur at much smaller scales (cm and below). Moreover, fast processes (e.g., high velocity flow in highly permeable rocks and fractures) and slow processes (e.g., flow in low permeable porous rocks) coexist, and they both have to be correctly represented to obtain reliable numerical simulations. Additionally, at the continuum (or Darcy) scale, porous media present highly heterogeneous permeability tensors which are a measure of the ability of the rock to conduct flow. Thus, accurate numerical models require very high resolution grids both in space and time to capture all the relevant physics of the phenomena. However, the large size of the domains and the large number of realizations needed to reduce uncertainty, make field scale simulations impracticable on such high resolution grids.

Traditionally, upscaling techniques were used to reduce the computational costs. These methods map the rock and the fluid propertiesdefined on a fine-scale grid to a coarser resolution. However, in presence of complex physics and in big ratio between the fine and the coarse resolution, the upscaled solution may be non-satisfactory. For this reason some advanced schemes that are able to employ higher resolution grids have been developed. An example are the dynamic local grid refinements (DLRG) methods that allow to use an adaptive grid resolution throughout the time dependent simulation. They need some pre-calculated upscaled static quantities and at each time step, when the grid resolution is determined, the linear system needs to be constructed. The well known multiscale methods are another example of advanced algorithms that combines different scales and speed-up simulations. Finally, the algebraic dynamic multilevel method (ADM) is able to use a dynamic adaptive grid in space. Thanks to the restriction and prolongation operators, it does not need to construct the discrete linear system at each time step but it is able to use an automatic (algebraic) procedure which constructs the linear system directly from the fine resolution. However, all these approaches use a fixed time resolution grid at each time step and are not combined with any advanced time integration scheme. On the other hand, the challenge of time adaptivity has been addressed by Adaptive Implicit methods (AIM), which operate with different levels of implicitness in different regions of the domain. This permits to apply the right scheme required to guarantee a stable solution. Another class of advanced schemes for the time complexity is the class of multirate methods. They allow for a flexible selection of time step size within the domain. In the early developments of multirate methods, the different time steps employed to integrate in time the sytem were selected apriori, exploiting the knowledge of the problem at hand. More recent extensions include a selfadjusting strategy to select the fast components in the system automatically. Note that most of the developments within the multirate literature address only ODE systems and hyperbolic wave equations.Furthermore, they mainly implement non-conservative procedures, which can lead to stability issues when applied to coupled flow-transport systems.

The aim of this thesis is the development of advanced techniques for the space-time discretization of multiphase flows in highly heterogeneous porous media simulation. These techniques are based on the adaptation of the computational grids, in space and in time, in order to optimize the trade-off between the accuracy of the solution and the efficiency of the simulation. We developed a new multirate technique able to preserve the mass conservation based on a flux partition strategy. Local mass conservation is a desired property for numerical methods that solve time dependent conservation laws to converge to the right weak solution. For the first time, we integrated this method into nonlinear multiphase flow simulators. We combined the time adaptivity, obtained by the multirate method, with different advanced techniques able to treat the spatial heterogeneity. First, we combined the multirate method with a multiscale approach. It solves the problem on a coarse grid capturing all the fine scale properties with the help of some ad hoc basis functions. Then, we integrated the multirate approach with an algebraic dynamic multilevel method for the solution of the transport equations. The obtained method is able to use different grid resolutions, both in space and time, based on an error estimator. In this way, the method employs the highest grid resolution only at the location of the moving saturation fronts.

DISCONTINUOUS GALERKIN APPROXIMATION OF FLOWS IN FRACTURED POROUS MEDIA ON POLYGONAL AND POLYHEDRAL GRIDS

Chiara Facciolà - Advisor: Prof. Paola F. Antonietti
Co-advisor: Prof. Marco Verani

Many Geophysical and Engineering applications, including, for example, fluid-structure interaction, crack and wave propagation problems, and flow in fractured porous media, are characterized by a strong complexity of the physical domain, possibly involving thousands of fault/fractures, heterogeneous media, moving geometries/interfaces and complex topographies. Whenever classical Finite-Element-based approaches are employed to discretize the underlying differential model, the process of mesh generation can represent the bottleneck of the whole simulation, as classical finite elements only support computational grids composed by tetrahedral/hexahedral/prismatic elements. To overcome this limitation, in the last decade a wide strand of literature focused on the design of numerical methods that support computational meshes composed of general polygonal and polyhedral (polytopic, for short) elements.
Within this framework, this thesis focuses on the problem of modelling Darcy's flow through a fractured porous medium and on its numerical approximation employing discontinuous Galerkin methods on polytopic grids (PolyDG). The problem of modelling this kind of flows has received increasing attention in the past decades, being fundamental in many energy or environmental Engineering applications, such as water resources management, oil migration tracing, isolation of radioactive waste and groundwater contamination, for example. Fractures are regions of the porous medium that are characterized both by a different porous structure and by a very small width compared to their length and to the size of the domain. The first feature implies that they have a very strong impact on the flow, since they can possibly act as barriers for the fluid (when they are filled with low permeable material), or as preferential paths (when their permeability is higher than that of the surrounding medium). The second feature entails the need for a very large number of elements for the discretization of the fracture layer and, consequently, a high computational cost. For this reason, one popular modelling choice consists in a reduction strategy, so that fractures are treated as (d-1)-dimensional interfaces between d-dimensional porous matrices, d=2,3. The flow in the porous medium (bulk) is then assumed to be governed by Darcy's law and a suitable reduced version of the law is formulated also on the surface modelling the fracture. Physically consistent coupling conditions are then added to account for the exchange of fluid between the fracture and the porous medium. Even if the use of this kind of dimensionally reduced models avoids the need for extremely refined grids inside the fracture domains, in realistic cases, the construction of a computational grid aligned with the fracture network is still a major issue. For example, a fractured oil reservoir can be cut by several thousands of fractures, which often intersect, create small angles or are nearly coincident. In line with the previous discussion, our aim is to take advantage of the intrinsic geometrical flexibility of PolyDG methods for the discretization of the coupled bulk-fracture problem. The inherited flexibility of DG methods in handling arbitrarily shaped, non-necessarily matching grids, and elementwise variable polynomial orders represents, in fact, the ideal setting to handle such kind of problems that typically feature a high-level of geometrical complexity. Another advantage is that the discontinuous nature of the solution at the matrix-fracture interface is intrinsically captured in the choice of the discrete spaces. Moreover, coupling conditions between bulk and fracture can be easily reformulated using jump and average operators (basic tools for the construction of DG methods) and then naturally embedded in the variational formulation, so that the coupling of the two problems can be efficiently handled.
For simplicity, in the first part of the thesis, we consider the case where the porous medium is cut by a single, non-immersed fracture. We consider all the possible combinations of primal/primal, mixed/primal, primal/mixed and mixed/mixed formulations for the Darcy's law describing the flow in the bulk and fracture, respectively. More precisely, a primal setting consists of having the pressure as only unknown. When dealing with the approximation of Darcy's flow, one may also resort to a mixed approach, where the flow is described through an additional unknown representing the (averaged) velocity of the fluid. This variable, often referred to as Darcy's velocity, is of primary interest in many engineering applications, so that the mixed setting is often preferred to the primal one, which may only return the velocity after post-processing the computed pressure, thus entailing a potential loss of accuracy. On the other hand, the primal approach is easier to solve, featuring a smaller number of degrees of freedom. For this reason, our aim is to design a unified setting where, according to the desired approximation properties of the model, one may resort to either a primal or mixed approximation for the problem in the bulk, as well as to a primal or mixed approximation for the problem in the fracture. In particular, the primal discretizations are obtained using the Symmetric Interior Penalty DG method, and the mixed discretizations using the Local DG method, both in their generalization to polytopic grids. We perform a unified analysis, based on the flux formulation, of all the derived combinations of DG discretizations, where the coupling conditions between bulk and fracture are imposed through a suitable definition of the numerical fluxes on the fracture faces. We prove well-posedness and derive a priori hp-error estimates in a suitable (mesh-dependent) energy norm. Next, we extend the primal-primal formulation to the case of networks of intersecting fractures, supplementing the model with conditions prescribing pressure continuity and flux conservation along the intersections. Both the bulk and fracture discretizations are obtained employing the SIPDG method extended to the polytopic setting, the key point to obtain a DG discretization being the generalization of the concepts of jump and average at the intersection. We prove the well-posedness of the discrete formulation and perform an error analysis obtaining a priori hp-error estimates. All our theoretical results are validated performing numerical tests with known analytical solution. Moreover, we consider more realistic configurations involving totally immersed networks of fractures. Finally, we briefly explore the case where the position of the fractures is uncertain and may be described by a stochastic parameter. We present some preliminary numerical results that employ a stochastic collocation approach.

MIXED-DIMENSIONAL PDES ARISING FROM MODEL REDUCTION FOR MULTISCALE MODELING OF VASCULAR TRANSPORT

Federica Laurino - Supervisor: Prof. Paolo Zunino

The main focus of this PhD project is the development a comprehensive model for nanoparticles transport within a realistic microvascular network and their interaction with the vessel wall. The interest is motevated by the fact that nanoparticles have been promoted as an efficient tool for the early diagnosis and treatment of multiple diseases, such as cancer. Indeed, in a comparison with conventional treatments, nanoparticle based therapies show several advantages; for example they allow the combination of multiple therapeutic agents and facilitate the protection of drugs from clearance by the immune system. However, the integration of nanomedicines into clinical practice is still challenging. The main interest is to understand particles transport across biological barriers to their final pathological targets and critical to this goal is understanding the dynamics of particles. In this context, there is a pressing need for mathematical models that can be used in alternative to in vivo models to guide the rational design of nanomedicines.

In this work develop a model, based on PDEs, able on one hand to accurately describe all the phenomena we are interested in combining different scales, on the other hand to handle complex realistic geometries with a reasonable computational cost. More precisely, we propose a model able 1. to describe at the macroscale the blood flow in a vascular network of arbitrary geometry, taking into account the non Newtonian rheology of the blood and other relevant effects, such as the lymphatic drainage; 2. to describe, still at the macroscopic level, the transport of particles in the vasculature; 3. to at the mesoscale, take into account the interactions between the particles and the vessel wall. Indeed, the particle surface can be covered by ligand molecules which interact with the receptors on the vessel wall, causing the closure of the bonds and consequently the adhesion of the particle; 4. to handle vascular networks with a very complex geometry with an affordable computational cost. To address the last point, our idea is to use a reduced model for the fluid dynamics and the transport in the vasculature. More precisely, we exploit the fact the the size of the blood vessels is small compared to the size of the surrounding interstitial volume and we approximate the 3D problem in the network with a 1D problem, keeping the information about the 3D geometry of the vessels in the imposition of the coupling conditions (Figure 1). Therefore, concerning the description of the blood flow in the vasculature and the interstitial flow in the extravascular space, we use a 3D-1D model based on the Darcy's law coupled the Poiseulle's law. The presence of the red blood cells is taken into account through the haematocrit variable. For the transport of injectable particles, we use a model based on the advection-diffusion-reaction equation, where the reaction term describes the adhesive behaviour of the particles. Indeed it involves the *adhesion parameter*, computed on the basis of a discrete model for the specific ligand-receptor adhesive interactions which combines the Lattice Boltzmann and the Immersed Boundary method. We also present some preliminary ideas and results about a possible extension of the model to take into account the capability of the particles to release the loaded agent in the extravascular space (Figure 2). Some parameters of the model, such as the diffusion coefficient characterizing the loaded agent, are predicted from experimental data using an error minimization algorithm.

n order to perform the approximation of the 3D problem in the microvasculature with a 1D model, we use a topological model reduction technique based on averaging.

We analyze this tecnique from the mathematical standpoint in the case of a simplified prototype problem. In particular, starting from a fully 3D system of elliptic PDEs coupled by mixed interface conditions, we identify the assumptions that are needed for the derivation of the reduced 3D-1D coupled problem. Moreover, we study the modeling error, i.e. the difference between the solution of the original and reduced problem, and we prove that for infinitesimally narrow inclusions the reduced model converges to the original one. In addition, we introduce an estimator of the model error to localize the error on the computational mesh once that the problem is discretized by the Finite Element Method (FEM). This could represent a first step for the development of adaptive methods, able to refine the grid where the error is larger, optimizing in this way the ratio between accuracy and computational cost of the simulation. We also investigate the efficiency of the available solvers for the reduced problem. The model reduction technique based on averaging sensibly changes the sparsity pattern of the matrix of the linear system arising from the FEM discretization of the problem and the presence of non local coupling terms affects the performance of the direct solvers. We investigate the efficiency of the Algebraic Multigrid method (AMG) applied to the problem at hand and the experiments show that the AMG is faster with respect to the direct solvers and it is robus with respect to the parameters of the problem.

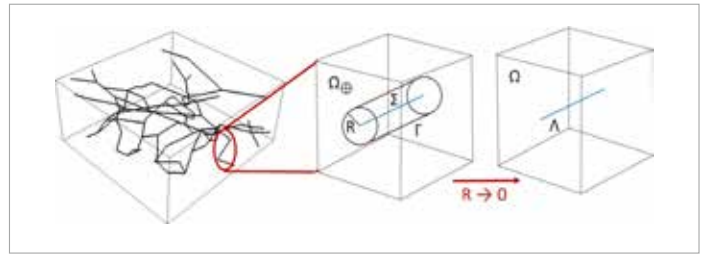

Fig. 1 - Reduction from 3D to 1D description of a vascular network.

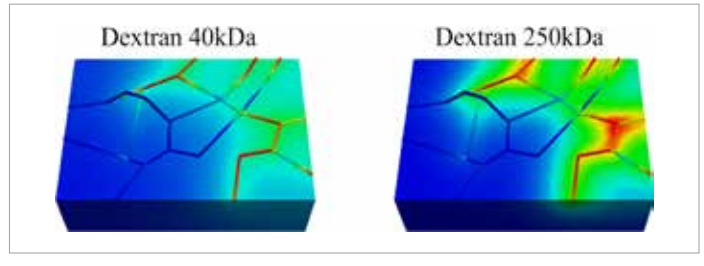

Fig. 2 - Density of adhering nanoparticles in the vascular network and concentration of released Dextran molecules with different molecular weight (40, 250 kDa) in the interstitial volume.

MATHEMATICAL MODELING AND MACHINE LEARNING FOR THE NUMERICAL SIMULATION OF CARDIAC ELECTROMECHANICS

Francesco Regazzoni - Advisor: Prof. Alfio Quarteroni

Co-advisor: Prof. Luca Dedè

Computer-based numerical simulations of the heart, also known as in silico cardiac models, are increasingly assuming a recognized role in the context of computational medicine and cardiology. However, the intrinsic multiscale nature of the cardiac activity, for which energy is consumed at the microscale by subcellular mechanisms to produce work at the macroscale for the whole organ, risks to harm the exploitation of computational medicine for the heart, as it raises a challenging trade-off between accuracy of the models and computational efficiency of numerical simulations.

In the past decades, several efforts have been dedicated to the construction of mathematical models describing the complex dynamics of *sarcomeres*, the fundamental contractile units of the cardiac muscle tissue. However, because of the intrinsic complexity of the phenomenon of force generation, huge computational costs are associated with the numerical approximation of such models. Despite several attempts to capture the fundamental mechanisms underlying the force generation phenomenon into a tractable number of equations, the existing organ-level cardiac mathematical models rely on two alternative strategies to describe microscopic force generation.

- Phenomenological models, whose parameters often lack a clear physical interpretation; moreover, the intrinsic difficulties in measuring sarcomeres under the conditions occurring during an heartbeat hamper the predictive power of such models.

- Biophysically detailed models, derived from physics first principles, whose numerical solution, because of their complexity, is typically obtained by means of a **Monte Carlo (MC) approximation**. However, the MC method is in fact inefficient: the simulation of a single heartbeat may require tens of hours of computational time on a single core.

In this thesis we develop a mathematical and numerical multiscale model of cardiac electromechanics, wherein the mechanisms of active force generation are described by means of new biophysically motivated models. To derive these subcellular models, we start from a detailed description of the microscopic mechanisms of regulation and activation of the contractile proteins. Then, we introduce suitable assumptions of conditional independence among the states of the stochastic processes associated with the protein dynamics. In this manner, we explicitly represent only the most relevant interactions among the proteins involved in the force generation process, while we neglect secondary interactions. This leads to accurate results — that we validate against experimental data — obtained with a drastic reduction of computational cost with respect to the models currently available in literature. Indeed, in virtue of the partial decoupling among the stochastic processes, the number of variables of our models is linear in the number of proteins, instead of exponential, as in the models available in the literature. As a matter of fact, the numerical solution of the models proposed in this thesis requires only a few seconds of computational time for an heartbeat on a single core.

As it is crucial to multiscale electromechanical modeling, we establish the link between the variables describing force generation at the microscale and those describing the strain and the stress of the tissue at the macroscale. This allows to couple, in a mathematically sound manner, the subcellular models proposed in this thesis — characterized by a stochastic behavior — with models for cardiac electrophysiology and for passive and active mechanics — based on a deterministic formalism — written as systems of Ordinary Differential Equations (ODEs) and Partial Differential Equations (PDEs).

In this thesis we also combine the proposed subcellular models with a newly developed Machine Learning algorithm, in order to speedup their numerical resolution in the multiscale electromechanical model. Specifically, a reduced model based on Artificial Neural Networks (ANNs) is trained from a collection of simulations generated by means of biophysically detailed force generation models. In this manner, the computationally demanding training phase can be performed offline, with the advantage of a huge speedup when the trained ANN-based model is exploited in replacement of the high-fidelity model used to generate the training data. We remark that the applicability of the Machine Learning algorithm proposed in this thesis overlooks the specific application to cardiac force generation models. Indeed, it can be generally applied to perform data-driven model order reduction (MOR) of high-fidelity models written as ODE or PDE systems, as we show through examples, or to perform data-driven modeling, by building a model directly from experimental data.

Overall, our multiscale model for cardiac electromechanics achieves an excellent balance between accuracy of the models, their rigorousness and computational efficiency in large-scale simulations.

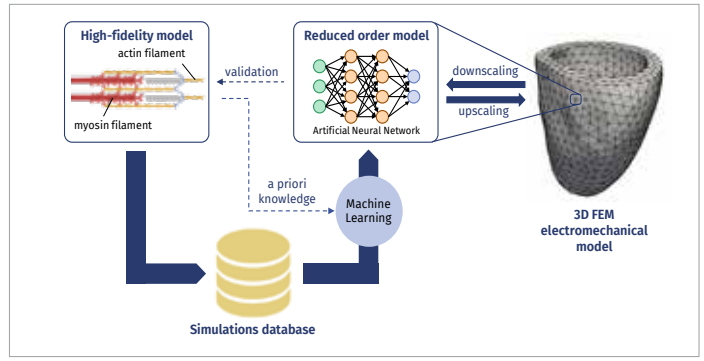

Fig. 1 - Machine-learning based strategy to speedup the numerical approximation of multiscale cardiac electromechanics.

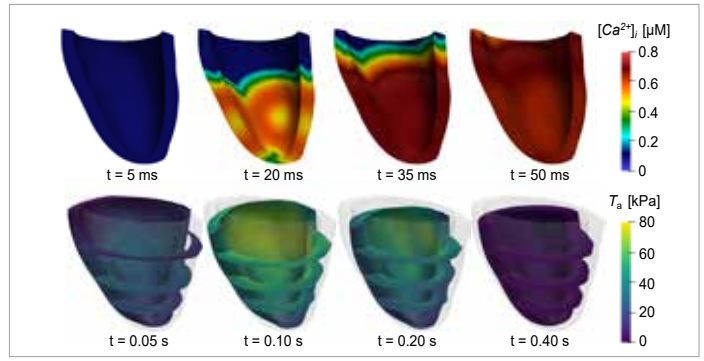

Fig. 2 - Propagation of the calcium signal (top) and consequent development of active tension (bottom).

GAME THEORY FOR IMPROVING MEDICAL DECISIONS AND MANAGING BIOLOGICAL SYSTEMS

Monica Salvioli - Supervisor: Prof. Roberto Lucchetti

Co-supervisor: Prof. Kateřina Staňková

While initially situations arising in economic and social sciences were the main focus of game theory, over the years it has been applied in fields as diverse as business, political science, philosophy, psychology and computer science as well as medicine and biology. The aim of this dissertation is to explore some opportunities for applying game theory to improve decision making in the medical field and the management of biological systems.

In Chapter 2 we use tools from classical non-cooperative game theory to model issues arising in the healthcare system. In particular, we model the relationship between doctors and patients in the context of clinical trials. Enrollment into clinical trials is fraught with common and conflicting interests, as doctors may lean toward offering the experimental treatment rather than the standard one outside randomization and, as a consequence, patients can be expected not to trust researchers. For the society, this is an extremely negative outcome, that can harm the whole process. Our work improves and refines previous models found in the literature. We examine possible countermeasures proposing an incentive mechanism and a signaling game to better understand the system and to shed light on the inherent conflicts and critical aspects. The goal is moving closer to the best collective outcome, meeting the need of system transparency and a full cooperation between doctors and patients.

While classical game theory typically considers players to be rational and evolutionary game theory considers players to be driven by natural selection, of crucial interest are interactions between rational agents (such as humans) and evolving biological systems (such as diseases, natural pests, bacterial systems, fisheries), which have not been formalized yet. This dissertation bridges this gap by introducing Stackelberg Evolutionary Games . Stackelberg evolutionary games resemble classical Stackelberg games in the sense that they maintain the same structure, where one player (usually called the “leader”) moves first and all the other player (usually called “follower”) react afterwards. In general, the leader takes advantage of this asymmetrical situation by anticipating the response of the follower and steering it to a desirable outcome for himself/herself. In this context the followers do not need to be a rational player, as commonly understood, but are represented by a population of evolving individuals. When the followers are evolving individuals, their dynamics conform more to evolutionary game theory and the response to the leader’s choice is represented by their eco-evolutionary dynamics. Thus, the game is played on two levels. On the first level, the followers are engaged in an eco-evolutionary game where ecological dynamics involve changes in population size and evolutionary dynamics determine changes in heritable traits. In these games the strategies of the players are inherited, and their payoffs take the form of increased survivorship. The solutions to this kind of games are evolutionarily stable strategies (ESS), i.e. strategies that when adopted by an entire population cannot be invaded by any rare alternative strategies. On a higher level, human players are involved in a traditional game. They do not inherit their strategies but choose them, and payoffs are given by a classical utility function. The solution of this kind of games is usually a Nash equilibrium, corresponding to no regret strategies. In classical game theory, it is well known that for the leader the Stackelberg solution is always at least as good and often much better than the Nash. This suggests that as humans we should be able to be leaders and anticipate the consequences of our decisions.

In this dissertation we present two applications. In Chapter 4 we apply Stackelberg Evolutionary Games to fisheries management. Fish populations subject to heavy exploitation are expected to evolve over time, usually reducing the average body size, with important consequences both from the ecological point of view and from the economic one. For this reason fisheries management should be adjusted to mitigate the potential negative effects of such evolutionary changes. We model fisheries management as a game of a manager versus a fish population, where the former can adjust the harvest effort and the net size in order to maximize profit, while the latter responds by evolving the size at maturation in order to maximize the fitness. We analyze three solution concepts: i) an ecologically enlightened approach (leading to Nash equilibrium in game-theoretical terms), where the manager acts without considering the evolutionary consequences of their actions; ii) an evolutionarily enlightened approach (leading to Stackelberg equilibrium in game-theoretical terms), where the manager anticipates and steers the evolution of the fish; iii) a domestication approach (leading to team optimum in game-theoretical terms), where the manager breeds the fish size selectively so that it maximizes manager’s profit.

In order to determine the impact that incorporating evolution into the management decision framework can have in terms of profit, we calculate the manager’s profit at both Nash and Stackelberg equilibria, compare those to the team optimum, and analyze how sensitive these equilibria are with respect to small variations of model parameters. As expected, domestication results in the largest size for the fish and the highest profit possible for the manager, obtained by mean of a low harvest effort and large net size. Moreover, we show that with the ecologically-enlightened (Nash) approach the manager tends to adopt a high harvest rate and a small net size that eventually leads the fish to be smaller. On the other hand, with the evolutionary-enlightened (Stackelberg) approach the manager scales back the harvest effort and selects a bigger net size: this entails sacrificing profit in the short term but will help to preserve both the size of the fish and the profit in the long term. Overall, our results suggest that mitigating evolution and maximizing the profit are not necessarily in conflict and encourage fish managers to take evolutionary dynamics of fish into account in order to prevent them from reducing in size and eventually to prevent the fish stocks from collapsing.

In Chapter 5 we apply the same theoretical framework to the treatment of metastatic cancer. Despite great strides in the prevention and in the treatment of early stages of cancer, for metastatic cancers cure is rarely possible. In fact, life expectancy of patients with metastatic cancer remains the same as 50 years ago. Current standard of care typically applies a drug at maximum tolerable dose, either continuously or at fixed intervals, and the same treatment regimen continues until there is unacceptable toxicity or tumor progression. While this approach is often initially successful, cancer cells usually evolve resistance leading to treatment failure. As the success of cancer cells depends on the therapeutic strategy, and the efficacy of treatment depends upon the resistance strategies of cancer cells, cancer treatment can be modeled as a Stackelberg Evolutionary Game, between the physician (rational leader) and the cancer cells (evolving follower). Through this lens, it is possible to explain mathematically how the standard of care in metastatic cancers promotes the evolution of therapy resistance and the subsequent treatment failure. Our results show that, by selecting the optimal dose, the physician can anticipate the cancer cells’ response to the treatment and steer their eco-evolutionary dynamics towards a more desirable outcome for the patient. This includes maintaining a stable tumor burden, reducing toxicity and postponing the evolution of resistance.

NUMERICAL METHODS FOR THE SIMULATION OF PARTICLE MOTION IN ELECTROMAGNETIC FIELDS

Abele Simona - Supervisor: Prof. Luca Bonaventura

Co-supervisors: Prof. Carlo De Falco, Prof. Sebastian Schöps

In this thesis we study several numerical methods for the approximate solution of problems arising in electromagnetism. The motivations for this study mainly came from the context of particle accelerators, but the proposed strategies have a much broader range of applications, as we will try to emphasize when presenting the proposed numerical techniques.

Particle accelerators are structures used to bring charged particles, which travel in bunches, to a specific speed, or energy, so that they can be used for different purposes in various research areas like health, material sciences and, especially, researches related to the fundamental models of subatomic physics. Depending on the specific application, there are different characteristics of the accelerator that determine its overall quality as, for example, the energy spread in each bunch of particles or the luminosity, which is a quantity related to the achieved collision rate. Among the various types of machines, we will consider high-energy circular accelerators for particle collisions and, in particular, an application related to the High Luminosity - Large Hadron Collider (HL-LHC) project, which is a foreseen update of the well-known LHC, hosted by CERN.
The study of the beam dynamics requires the computation of several particle trajectories for a large number of revolutions. It is therefore crucial to obtain efficient numerical methods to describe the effect of each element in the accelerator over the charged particles.

Among the different elements, we will first consider the magnetic quadrupoles, which are used to focus particle beams and whose action on charged particles can be approximately described using a linear map that relates the positions and momenta at the inlet with the ones at the outlet. However, in the case of the design of large aperture quadrupoles foreseen for the HL-LHC project, more accurate methods are required. In this context, a wide class of methods relies on the so called transfer maps, which are functions that represent a nonlinear relation between position and momenta of the particles at the inlet and the outlet, respectively. These methods typically introduce strong approximations on the field shape, such as the so called Hard Edge model, that allow then to obtain an analytic expression for the transfer map. An example of such a method, that does not consider the dependence of the field on the axial coordinate along the magnet, is the so called thin model, used in SixTrack, the code developed at CERN to simulate the whole accelerator. Other methods are based on numerical integration of the Hamilton equations which describe the motion of the particles through the field. These methods are often computationally expensive and they are strongly affected by the discretization of the field. In particular, a standard way of representing the field is achieved by locally interpolating a set of sampled values on a 3D grid. A more efficient representation can be instead obtained directly from the solution of Maxwell equations. An example of these representations, which will be applied in this thesis, relies on the use of the so called generalized gradients. Once a field description is obtained, a wide range of techniques for the integration of Hamilton equations can be applied. In this thesis, we will compare the efficiency of explicit high order Lie methods, implicit high order symplectic integrators and more conventional, non-symplectic explicit Runge-Kutta methods. Using the field description based on the generalized gradients, we will also consider a specific gauge transformation that allows to reduce the computational cost of each vector potential evaluation by approximately 50%.
We finally compare the description of the nonlinear effects provided by our code with that computed with the thin model used in the SixTrack code.

Another important aspect is the numerical solution of Maxwell equations, which can provide the field description for design purposes or, as already mentioned, for the computation of particle trajectories. In this framework, the standard technique is the Finite Element Method, but other alternatives, such as the Boundary Element Method or IsoGeometric Analysis (IGA), are available and offer important advantages in specific situations. Among the different types of geometries on which we could solve Maxwell equations, we will consider axisymmetric domains which often arise in electromagnetic problems and, in particular, in particle accelerators. The solution of Maxwell equations in axisymmetric domains has been studied by different authors, both in the case of axisymmetric fields and for the general case. In this thesis, we present a method combining a spectral Fourier approach and (IGA). The resulting approach has good approximation properties and its discrete approximation spaces form a de Rham complex. This allows for an accurate and efficient representation of the electromagnetic field and for the solution of a wide range of problems in electromagnetism. In particular, we will apply this method to the computation of resonant modes in accelerating TESLA cavites which are another type of element used in particle accelerators often made of superconducting materials. They are used to accelerate the charged particles by means of an oscillating electric field, which is produced in such a way that the particle travelling through the cavity experiences always an accelerating force. The oscillating frequency of the electric field depends, of course, on the particle speed and it should match the principal resonant mode of the cavity. For this reason it is important to accurately compute the resonant frequencies of a cavity, which amounts of solving an eigenvalue problem with suitable boundary conditions. A TESLA cavity with the electric field is depicted in Figure 1.

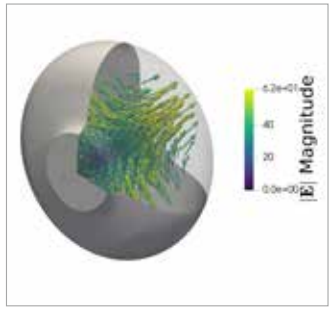

Fig. 1

Finally, we consider the problem of reconstructing the field from a realistic measurement process.
In this case, the noise level affects the accuracy of the result. On the other hand, if the data are produced by a numerical model, the main source of uncertainty is related to the model parameters. More specifically, we consider the reconstruction of a quadrupolar field from the voltages induced on a spinning coil, which shows that the proposed method can also be applied to the reconstruction of the magnetic field from the available measurements. Apart from the uncertainty quantification procedure, the reconstruction might be useful to obtain a field description of those magnets for which a numerical model does not exist or to improve an existing one through an hybrid approach that combines the mathematical model with the measured data.

STEADY NAVIER-STOKES EQUATIONS IN DOMAINS WITH OBSTACLE AND APPLICATIONS TO THE STABILITY OF SUSPENSION BRIDGES

Gianmarco Sperone – Supervisor: Prof. Filippo Gazzola

Vortex shedding is the phenomenon observed when a fluid hits a rigid structure immersed in the fluid, causing the formation of vortices on the downwind side of the obstacle. Indeed, vortices are shed alternately from one side to the other giving rise to the *von Kármán vortex street*, as displayed in Figure 1. Shifting low-pressure zones are then created on the leeward side of the structure which, in turn, generate a fluctuating force that acts orthogonally to the flow direction; we shall refer to this force as the *lift*. When the structure considered is the deck of a suspension bridge and the fluid flow is the wind, a consequence of the von Kármán vortex street is the oscillating movement of the deck towards the low-pressure zone, a tremor known in the literature as *vortex-induced vibration*.

Naturally, if the input of energy from the wind into the deck grows unsteadily, violent lift forces will appear, possibly leading to the collapse of the suspension bridge. In this chaotic situation, the whole structure oscillates and both the cables and the hangers generate unexpected behaviors of the deck, such as torsional movements. The main general goal of the present research is to understand, analyze and quantify (in a suitable manner) the existing relationship between the fluid velocity, the resulting lift, and, ultimately, the attainment of the thresholds for hanger slackening and cable shortening.

This thesis is organized as follows. The Introduction (Chapter I) serves as a summary, where we survey some of the existing (and sometimes contradictory) results on turbulence, fluids and structures, and suggest several natural questions whose answers would increase the mathematical understanding of these phenomena. In Chapter II we focus our attention on the structure: the Melan equation for suspension bridges is derived by assuming small displacements of the deck and inextensible hangers. We determine the thresholds for the validity of the Melan equation when the hangers slacken, thereby violating the inextensibility assumption. To this end, we preliminarily study the possible shortening of the cables: it turns out that there is a striking difference between even and odd vibrating modes since the former never shorten. These problems are studied both on beams and plates. For the remaining parts of this work we analyze exclusively the hydrodynamic component of the fluid-structure interaction problem considered. In Chapter III, a variational formulation for a class of mixed and nonstandard boundary conditions (based on the vorticity, pressure, normal and tangential components of the velocity field) on a smooth obstacle is discussed for the Stokes equations. Possible boundary data are then derived through separation of variables of biharmonic equations in a planar region having an internal concave corner. Explicit singular solutions show that, at least qualitatively, these conditions are able to reproduce vortices over the leeward wall of the obstacle. Then, Chapter IV is devoted to the study of planar viscous flows governed by the stationary Navier-Stokes equations with inhomogeneous Dirichlet boundary data in non simply connected domains. In a symmetric framework the appearance of forces is strictly related to non-uniqueness of the solution. Explicit bounds on the data ensuring uniqueness are then sought and several functional inequalities (concerning relative capacity, Sobolev embedding, the continuity constant of the Bogovskii operator) are analyzed in detail: explicit bounds are obtained. The case of ``almost symmetric'' frameworks is also considered. An explicit universal threshold on the Reynolds number ensuring that the flow generates no lift is obtained regardless of the shape and the nature of the obstacle. A shape optimization problem, aiming to minimize the impact of forces, is then addressed numerically. Connections of the results with elasticity and mechanics are also emphasized. Finally, several concluding remarks, open problems and future perspectives are the main content of Chapter V.

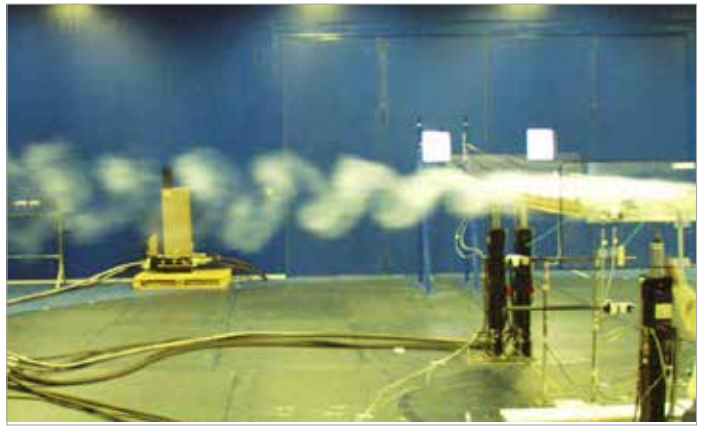

Fig. 1 - vortices around a plate in wind tunnel experiments at the Politecnico di Milano.