DOCTORAL PROGRAM IN MATHEMATICAL MODELS AND METHODS IN ENGINEERING

Chair:
Prof. Paolo Biscari

Equations are everywhere! Between the atmosphere and the wing of a spaceship, in the blood flowing in an artery, on the demarcation line between ice and water at the poles, in the motion of the tides, in the charge density of a semiconductor, in the compression algorithms of a signal sending images from outer space. Such equations represent real problems. The Mathematical Engineer can see and understand the nature of these equations, and can develop models in order to understand their relevant qualities and solve real problems.
This PhD program aims at training young researchers by providing them with a strong mathematical background and with ability to apply their knowledge to the solution of real-world problems that arise in various areas of science, technology, industry, finance, management, whenever advanced methods are required in analysis, design, planning, decision and control activities. PhD students carry our their research both in the development of new mathematical methods and in the implementation and improvement of advanced techniques in connection with specific contexts and applications.
The Faculty of the PhD program is responsible for the organization of the training and research activities of the PhD students. Decisions of the Faculty comply with the requirements and standards of the Doctoral School of the Politecnico di Milano. A Chairman is elected within the Faculty, for representative and coordination activities. Admission of students to the PhD program is decided after examination of the candidates. Students applying to our program must provide their CV, along with reference and motivation letters. After admission, each student is assigned a tutor. The tutor is a member of the Faculty who assists the student in the early stages of his career, especially in the choice of the courses and in identifying a thesis advisor.

The overall activity of the PhD students can be quantified in 180 credits. The PhD program has a duration of three years. Activity can be classified into:
introductory courses (no minimum number of credits required); main courses (at least 30 credits); specialized research training, including seminars, tutoring activity, participation to workshops/conferences, and scientific publications (at least 30 credits); development of a doctoral thesis (at least 90 credits).

At the end of each academic year, the PhD students report to the Faculty about their activity. The students report about attendance of courses and exams (and the corresponding grades), participation in various scientific activities (seminars, conferences, summer schools etc.), planning and intermediate results on their research project and preparation of the PhD thesis, and any other relevant activity. At the annual meeting the students also receive a grade by the Faculty. A negative grade may entail repetition of the current year of doctoral study (with suspension of the grant, if any) or exclusion from the PhD program, depending on the Faculty's decision. Mobility of PhD students to other institutions is strongly encouraged and financial support is provided to this purpose.

Among others, let us mention some typical types of professional skills and possible occupations of the graduated Doctors: analytic and numerical treatment of differential models for physical and industrial problems, quantitative methods in finance and risk management, operations research and optimisation, statistical modelling and data analysis.

Placement of graduated Doctors is expected in the following positions: research and development divisions of businesses, businesses involved in innovative design activities, financial institutions such as banks or insurance companies, public or private research centres, public and/or governmental agencies for social, economical, scientific study, planning or evaluation, Universities.

Since the PhD program in Mathematical Models and Methods in Engineering (formerly, Mathematical Engineering) has been active since the year 2001, we expect that a larger number of institutions and businesses will soon become more and more aware of the professional skills and expertise of graduated doctors.

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A PARTICLE FILTER APPROACH TO PARAMETER ESTIMATION IN STOCHASTIC VOLATILITY MODELS WITH JUMPS FOR CRUDE OIL MARKET

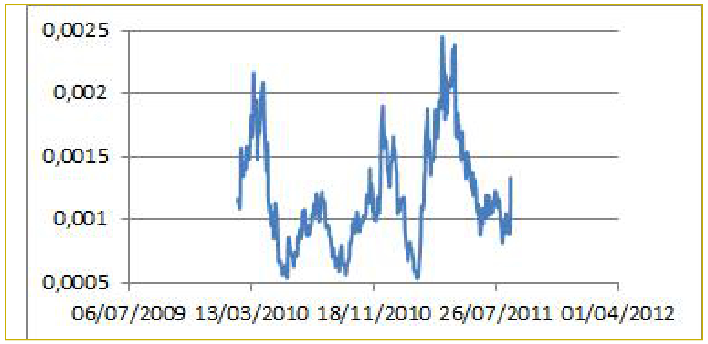
Gaetano Fileccia - Supervisor: **Carlo Sgarra**

The purpose of the research underlying the dissertation is to properly model the WTI crude oil market. To reach this objective two issues need to be addressed. The first issue is defining an appropriate modelization both for the processes observable in the market and for the latent processes driving the WTI dynamics; the second issue is designing and implementing an efficient inference algorithm for the model proposed.
Since the data available for our study come from the spot market and the futures prices market, the model choice will need to be performed according to satisfy the requirement to catch both the markets structures. Hence the choice to consider in models proposed a variance latent process (like in well known Heston model) besides the usual convenience yield process, typical of the storable commodity financial markets. After the financial theory, that traces the evolution of oil modeling, have been recalled, we proposed different models to be compared in catching the observed data time series. The different models are selected in order to resume the different possible modeling characteristics of commodity markets, as the presence of seasonality or the possibility to observe jumps in spot dynamics. These models, always belonging to the family of the affine models, to ensure admitting the affine futures price closed formula, have been tested and compared. Since the data comes from two different financial world, the spot market and the futures market, we needed to analyze our model for two different measures, under which we observed the two different sources of data: the historical measure for spot data and the risk neutral measure for futures prices. A definition of risk premia to allow switching from one measure to another (according to Girsanov theorem) have been provided besides the description of any model proposed. In this thesis several affine models have been presented and tested with data observed in WTI spot market and WTI futures markets. The main models proposed are three factor model with two latent processes (variance, V, and convenience yield) modeled via a CIR dynamics. Two models were considered one including a jump process, J, in the log-spot dynamics, x, and one neglecting it.

$$\begin{cases} dx_t = (r_f + c - \delta_t - \mu_J^*)dt + \sqrt{V_t}\,dW_{S_t} + dJ_t \\ dV_t = \beta(\bar{V} - V_t)dt + \xi\sqrt{V_t}\,dW_{V_t} \\ d\delta_t = \alpha(\bar{\delta} - \delta_t)dt + \sigma\sqrt{\delta_t}\,dW_{\delta_t} \\ dW_{S_t}\,dW_{V_t} = \varrho_V dt \\ dW_{S_t}\,dW_{\delta_t} = \varrho_\delta dt \end{cases}$$

The data used for the analysis are relative to WTI Cushing Crude Oil spot and futures quotations on NYMEX market from 11/01/2010 to 28/09/2011. We used the the first 85% if the data-set to get the inference on our model and reserved the last 15% of dates for out of the sample performances study. Besides spot data, for any working day we recorded a panel of 12 future contracts. Future contracts are usually characterized by their behaviour when time to maturity goes to zero: if, at a certain date, the futures quotation increases when the time to maturity become longer, it usually said the future market is normal, otherwise is said inverted.
The inference for these models have been run within the family of Bayesian technique. More specifically, the algorithm engine to carry out the inference comes from a recent evolution in the Markov chain Monte Carlo (MCMC) and link together the two bayesian techniques which had greatest relevance in the recent stochastic volatility econometrics literature: MCMC and Particle Filters. The algorithm family is thoroughly discussed, and main alternatives presented. Then the algorithm has been implemented to get inference both for the parameter set and the latent processes values. In the following figures are summarized the latent process estimate for the model presented: in the following figures are shown the convenience yield processes for both the model without and with jumps (respectively on the top and bottom row).
It is interesting to notice when the filtered process for the convenience yield is above a certain "threshold" representing the estimated cost of storage plus the interest rate (in the period considered the rate quoted by FED was at 0.25% on yearly basis): in this cases it is more valuable to keep the commodity sooner, since the yield produced is positive, hence in these cases the future prices are generally higher for shorter maturities than longer (inverted future market). It is possible to notice that both for the two proposed model the net convenience yield is greater than this threshold just in a few dates at the end of the analyzed period, when effectively the market showed an inverted structure, in the other dates, when the market was characterized by a normal future structure.
Other modeling alternatives have been considered, testing for seasonality, jumps and different correlation structures. Results were compared with a the benchmark model proposed by Liu and Tang in 2011. It resulted that adding a seasonality term does not affect significantly the model performances. In both correlation structure cases (double correlated and single correlated) the models including a volatility process factor display better performances when the spot dynamics is allowed to include jump activity, showing better results than all the other discussed models, included the benchmark model.
Resuming the results got for the time series analyzed, the models with volatility factor performed better than Liu and Tang model in describing the spot dynamics, as it is possible to notice analyzing the spot residuals; moreover, jump activity, absorbing part of the variance of the spot dynamics, has a positive impact in the whole model performances, thus resulted in normally distributed residuals.

1. Convenience yield (daily basis) for dynamics allowing jumps in spot process



2. Convenience yield (daily basis) for spot dynamics not allowed to jump

COMPLIANCE OPTIMIZATION FOR THIN ELASTIC STRUCTURES

Ilaria Lucardesi - Supervisors: **Ilaria Fragalà, Guy Bouchitté**

The main topic of the Thesis is *optimization of the compliance of thin elastic structures*. The problem consists in finding the optimal configurations that minimize the compliance, when an infinitesimal amount of elastic material is subjected to a fixed external load, and confined within a region having infinitesimal volume.
In particular, we treat the case of thin elastic bars, namely the problem in which the design region is a cylinder with infinitesimal cross section. The approach we adopt draws inspiration from some recent works by I. Fragalà, G. Bouchitté and P. Seppecher, in which they deal with the case of thin elastic plates, in which the design region can be approximated by a cylinder with infinitesimal thickness [G. Bouchitté, I. Fragalà, P.Seppecher: *Structural optimization of thin plates: the three dimensional approach,* Arch. Rat. Mech. Anal. (2011)].
We point out that these two problems are not merely technical variants one of the other, due to the substantial difference between the limit passages 3d-1d and 3d-2d, namely from 3 to 1 and from 3 to 2 dimensions.
The mathematical model reads as a family of variational problems depending on two parameters which describe the vanishing cross section of the design region and the vanishing filling ratio between the volume of the elastic material and the volume of the design region. We carry out the asymptotic analysis of the shape optimization problem in a double limit process. Moreover we derive a limit rod model written in different equivalent formulations, for which we are able to give necessary and sufficient conditions characterizing optimal configurations. Eventually we show that in pure torsion regime, for a convex design region and for very small volume fractions, the optimal shape tends to concentrate section by section near the boundary of the Cheeger set of the design. For more general loads, due to the interplay between bending and torsion, we are led to a more complicated model, related to some interesting variants of the Cheeger problem.

The study of optimal configurations led us to face another interesting variational problem: actually to establish whether homogenization phenomena occur in bars in pure torsion regime turns out to be equivalent to solve a *nonstandard free boundary problem* in the plane. This new problem is very challenging and, besides the link with torsion rods, it has mathematical interest in itself.
We study a 2d-variational problem, in which the cost functional is an integral depending on the gradient through a convex but *not strictly convex* integrand, and the admissible functions have zero gradient on the complement of a given domain $D \subset R^2$.
We are interested in establishing whether solutions exist whose gradient "avoids" the region of non-strict convexity of the integrand. Actually, the answer to this question is related to establishing whether homogenization phenomena occur in optimal thin torsion rods.
We provide some existence results for different geometries of D, and we study the nonstandard free boundary problem with a gradient obstacle, which is obtained through the optimality conditions.
We point out that achieving a complete characterization of domains D admitting such solutions remains by now open and is likely related to the regularity of ∂D and also to whether or not coincides with its Cheeger set.

One of the tools which can be employed to attack the problem of characterizing such domains is *shape derivatives for minima of integral functionals*.

The theory of shape derivatives is a widely studied topic (see *e.g.* the monograph by A. Henrot and M.Pierre *Variations et Optimisation de Formes. Une Analyse Géométrique*, Springer Berlin (2005), and the references therein), but the approach we propose is new and relies on assumptions which are weaker than the classical ones.
For Ω varying among open bounded sets in R^n with a Lipschitz boundary $\partial\Omega$, we consider shape functionals J (Ω) defined as the infimum over a Sobolev space of an integral energy of the kind

$$\int_\Omega [f(\nabla u)+g(u)]dx$$

, under Dirichlet or Neumann conditions on $\partial\Omega$.
Here f and g are given integrands, which are assumed to be continuous, convex, and to satisfy growth conditions, of order p and q respectively.
We prove that, when a given domain is deformed into a one-parameter family of domains Ω_ε through a smooth initial velocity field V, the corresponding shape derivative of J at Ω in the direction of V exists. We point out that, due to the lack of differentiability of the integrands, the computation of the shape derivative of J at Ω is not covered by the above quoted literature. Under some further regularity assumptions, we show that the shape derivative can be represented as a boundary integral depending linearly on the normal component of V on $\partial\Omega$. Our approach to obtain the shape derivative is new, and it is based on the joint use of Convex Analysis and Γ-convergence techniques. It allows to deduce, as a companion result, a new necessary condition of optimality for classical minimization problems in the Calculus of Variations.

The Thesis is organized as follows.
In the first part we gather the preliminaries: we recall the main mathematical tools of Convex Analysis, Geometric Measure Theory and Γ-convergence that we use in the Thesis, then we summarize the theory of linear elasticity, which motivates the study of the main problem.
The second part is devoted to the study of the compliance optimization problem in thin rods. The third part is dedicated to the above mentioned related problems.
When needed, the more technical proofs, usually concerning auxiliary lemmas or easy to prove statements, are postponed to the end of the Chapters, in the Appendix. The open problems and the possible advances are gathered in the Perspectives.

The chapters correspond to the following papers that have been written in these three years of PhD:

- G. Bouchitté, I. Fragalà, I. Lucardesi, P. Seppecher: Optimal Thin Torsion Rods and Cheeger Sets, *SIAM J. Math. Anal.*, **44**, 483-512, (2012);
- I. Lucardesi: Optimal design in thin rods, in preparation;
- J. Alibert, G. Bouchitté, I. Fragalà, I. Lucardesi: A non standard free boundary problem arising in shape optimization of thin torsion rods, to appear in *Interfaces and Free Boundaries* (2013);
- G. Bouchitté, I. Fragalà, I. Lucardesi: Shape derivatives for minima of integral functionals, in preparation.

POLYNOMIAL APPROXIMATION BY MEANS OF THE RANDOM DISCRETE

L2 projection and application to inverse problems for PDE with stochastic data

Giovanni Migliorati - Supervisor: **Fabio Nobile**

The main topic of the thesis concerns the polynomial approximation of aleatory functions by means of the random discrete L2 projection (hereafter RDP), and its application to inverse problems for Partial Differential Equations (PDEs) with stochastic data.
The motivations come from the parametric approximation of the solution to PDE models and its application to Uncertainty Quantification (UQ) in Engineering. In the literature, a lot of contributions are present on the analysis of the RDP in the noisy framework, and the field of nonparametric regression covers this subject in detail.
In the noiseless case, a complete theoretical analysis of the stability and convergence of the RDP was missing.
The interest towards the noiseless framework arises from the development of efficient techniques to deal with computational models expressed in parametric form. In the applications, the uncertainty often affects the measurements of the input parameters, which can be treated as random variables. Then it is reasonable to assume that the solution to the model can be evaluated without any additional noise, since the effects of deterministic approximation-type errors such as round-off errors are negligible w.r.t. the effects due to measurement errors.

Polynomial approximation by means of the random discrete L2 projection with noiseless evaluations

The former part of the thesis is devoted to the theoretical study of the stability and optimality property of the approximation methodology based on the RDP, with a focus on polynomial-type approximations of smooth functions. In the last years, increasing attention in the UQ community has been dedicated to non-intrusive approaches like RDP and Stochastic Collocation, in contrast to intrusive methods like Stochastic Galerkin. The RDP preserves the feature of being non-intrusive, still resorting to a global projection in the probability space.
This approach is particularly suited to the application to PDEs with stochastic data, being the evaluations of the computational model mutually uncoupled. In the Stochastic Collocation method based on Sparse Grids, the computational cost required to compute the optimal sparse grid becomes unaffordable as the dimension of the parameter set increases. To alleviate this effect, Adaptive Sparse Grids or greedy approaches to build quasi-optimal sparse grids might be considered, but their application remains confined only to moderately high dimension of the parameter set. On the other hand, the analysis in this thesis reveals that the stability of the RDP improves as the dimension of the parameter set increases, making it a tool naturally oriented towards high dimensional problems. A flexible and easy incorporation of new measurements to improve the current accuracy of the polynomial approximation is another point of strength of the RDP versus the Stochastic Collocation method, since the new measurements do not have to satisfy any hierarchical compatibility.
In Chapter 1, the RDP on polynomial spaces is presented, as a tool to accurately approximate a multivariate function depending on a random variable distributed according to a given probability density. The stability and optimality of the approximation error evaluated in the L2 weighted norm are addressed, under the assumption that the density is bounded away from zero. In this analysis, the main result achieved is a monovariate probabilistic optimal convergence estimate with the uniform distribution, provided the number of evaluations scales as the square of the dimension of the polynomial space. Several numerical examples confirm the theoretical results, with aleatory functions defined on parameter sets featuring low to moderately high dimension. The role of smoothness of the target function has been investigated as well. In Chapter 2 the proof of the stability and optimality in expectation of the RDP is extended to any monotone set of multi-indices identifying the polynomial space, and to any dimension of the parameter set. For a specific class of PDE models, that includes the elliptic model and the linear elasticity model, an exponential convergence estimate w.r.t. the number of sampling points has been proved, with a priori optimal choice of the polynomial space. This estimate clarifies the dependence on the dimension of the parameter set, and establishes a relation between the convergence rate of the RDP and the convergence rate of the classical Stochastic Galerkin method. Afterwards, in Chapter 3 the analysis of the RDP is extended to more general densities, focusing on how the choice of the density affects the optimal convergence rate. It is shown that the assumption on the density being bounded away from zero is strictly required in the proof of the optimal convergence theorem. The methodology based on the RDP is then applied in Chapter 4 to approximate Quantities of Interest related to the solution to Partial Differential Equation models with stochastic data. Several examples are presented, featuring the Darcy model, the linear elasticity model and with Navier-Stokes equations, with values of the coefficient and geometry of the inclusions governed by random variables.

Application of the Factorization Method to Electrical Impedance Tomography in inhomogeneous uncertain background

In the latter part of the thesis, the methodology previously developed for the forward problem is applied to inverse problems for PDEs with stochastic coefficients.
In Chapter 5 the problem of Electrical Impedance Tomography (EIT) is introduced. The contribution of this thesis concerns the numerical analysis of the Factorization Method applied to the Continuous Model in EIT, in the case of inhomogeneous uncertain background. The Factorization Method is an imaging technique belonging to the class of Qualitative Methods. It aims to detect the presence and location of inclusions inside the computational domain, exploiting the information obtained when observing the solution to the associated PDE model on the whole boundary of the domain. A numerical scheme to solve the dipole-like Neumann problem in inhomogeneous background has been proposed. This allowed to extend the range of application of the Factorization Method to inhomogeneous deterministic background, i.e. to background diffusion coefficient featuring a nonlinear spatial dependence on the spatial coordinates, as well as to the case of inhomogeneous uncertain background. More specific types of background (i.e. piecewise constant in a partition of unity of the computational domain) are then considered. The range of variation of the background coefficient has been investigated up to two orders of magnitude. In addition, a Tikhonov regularization technique embedding Morozov principle has been compared with another regularization technique proposed in the literature and based on the Picard Criterion. The effectiveness of the proposed approach has been checked also in presence of noise contaminating the measurements of the observation operator. When uncertainty in the background coefficient is present, a classification has been proposed depending on how the measurements of the random observation operator are collected. Two situations are distinguished: the case of arbitrary measurements and the case of paired measurements. Accordingly, three variants of the Factorization Method have been proposed to cope with the uncertainty in the background, and their capabilities have been presented in an extensive set of numerical tests. In Chapter 6 the variants of the Factorization Method proposed in the previous chapter are accelerated by means of the RDP, exploiting the techniques that have been presented in the previous part of the thesis. A convergence analysis of the approximation error committed by the RDP has been proposed, using suitable norms to control the error between the spectrum of the original operator and the spectrum of the polynomial surrogate model.

COVARIANCE OPERATORS AS OBJECT DATA: STATISTICAL METHODS AND APPLICATIONS

Davide Pigoli - Supervisor: **Piercesare Secchi**

This thesis is part of a line of research which deals with the statistical analysis of data belonging to a non Euclidean space. In recent years, attention to the statistical analysis of non Euclidean data has been growing. The conceptual framework is that of Object Oriented Data Analysis. This approach focuses on the atoms of the statistical analysis. While this is usually a number or a vector, new technologies have provided different kinds of data, such as high dimensional arrays, curves, shapes, diffusion tensors… Many of these can be seen as elements of a non Euclidean space. Indeed, non Euclidean data are mathematical objects more complex than numbers or vectors and they do not belong to a linear space. Thus, even the most simple statistical operations, such as finding a centerpoint for the data distribution or evaluating variability about this center, represent a challenge. Statistical analysis needs to carefully consider the mathematical properties of the data at hand and consequently to reformulate traditional methods in this new setting.
Data belonging to a Riemannian manifold are particularly interesting both from a mathematical and from a practical point of view. Studies in this field have been motivated by many applications: for example Shape Analysis, Diffusion Tensor Imaging and estimation of covariance structures. The general aim of these studies is the extension to Riemannian data of traditional statistical methods developed for Euclidean data, such as point estimation of mean and variance, exploratory data analysis, dimensional reduction, testing hypothesis among different populations and smoothing. The common idea is to find the correct distance to compare two elements in the non Euclidean space and to build statistical methods based on that distance.
This thesis proposes a twofold contribution in this research field. First, we consider the generalization of the above methods to infinite dimensional covariance operators. This problem arises within Functional Data Analysis, where the interest is on the second order structure of the functional random process. Under non restrictive assumption on the random process which generates the data, the covariance operator is a trace class operator with non negative eigenvalues. Thus, it belongs to an infinite dimensional non Euclidean space. Distances for comparing positive definite covariance matrices are either extended or shown to be inapplicable for functional data. In particular, an infinite dimensional analogue of the Procrustes size and shape distance is developed. The proposed distances are used to address important inferential problems, namely, the point estimation of covariance operators and the comparison of covariance operators between two populations of curves.
A second contribution lies in the introduction of spatial dependence among Riemannian data, with a particular attention to the case of covariance matrices. It is not a trivial problem to define stochastic dependence in non linear spaces. We consider both the modeling of the dependence on the manifold, generalizing the definition of covariance in linear spaces through the expected values of square distances, and the possibility to approximate non Euclidean data in the appropriate tangent space, where traditional statistical techniques can be used. First, the Riemannian semivariogram of a field of covariance matrices is defined. Then, we propose an estimator for the mean which considers both the non Euclidean nature of the data and their spatial correlation. Simulated data are used to evaluate the performance of the proposed estimator: taking into account spatial dependence leads to better estimates when observations are irregularly spaced in the region of interest. Finally, a kriging estimator based on a tangent space model is proposed for covariance fields. This allows to deal with non stationary fields, the deterministic drift being handled in the tangent space with traditional spatial statistics techniques.
The choice to address these specific problems is guided both by their mathematical and statistical interest and by the considered applications. Indeed, we illustrate two problems where the analysis of covariance operators of functional random variables brings insight to the statistical analysis. The first one is the comparison of the covariance functions between different groups of patients of the well known AneuRisk dataset (http://mox.polimi.it/it/progetti/aneurisk), aiming at supporting some choices of the previously published analysis. The second analysis addresses a linguistic problem, namely the exploration of relationships among Romance languages. Here, the statistical units are the covariance structures among frequencies for speakers of different languages. Data come from speech recordings provided by Prof. J. Coleman (Phonetic Laboratory, University of Oxford) and preprocessed by P.Z. Hadjipantelis (University of Warwick). Linguistic scholars suppose that the covariance operator of frequency intensities catches significant phonetic features of the language. Using a distance-based approach to analyse these covariance operators for some Romance languages, preliminary results support this hypothesis, while also detecting some phonetic structures which deserve deeper linguistic explorations.
A meteorological problem is the most immediate application for spatial statistics for Riemannian data. In particular, we illustrate the analysis of covariance matrices between temperature and precipitations measured in different stations in Quebec, Canada. We show that taking into account spatial dependence provides estimates that are in a better agreement with known meteorological information.